Chapter 1

Temperature measurements

The temperature is related to the molecular kinetic energy of a body. Various definitions of temperature have been proposed: for example, it has been defined as the condition of a body by which heat is transferred to and from other bodies. From the point of view of measurement, the basic concept is that temperature is an index of molecular activity. A change in body temperature has several effects:

- changes in physical state;
- changes in chemical state;
- changes in physical dimensions;
- changes in electrical properties;
- changes in radiation properties.

Usually, the last three effects are exploited for temperature measurement. Different temperature scales are used, the most common are:

- centigrade scale, measured in Celsius degrees, ^oC;[1]
- absolute scale, measured in Kelvin degrees, ^oK;
- Fahrenheit scale, measured in Fahrenheit degrees, ^oF.[2]

1.1 Thermometers

1.1.1 Thermometer, liquid-filled

They consist of a tank, bulb, which is connected to a capillary (with diameter of approximately one tenth of a millimetre) attached to a graduated scale. Mercury or alcohol can generally be used. On the one hand, the latter has the advantage of a higher expansion coefficient than the former. On the other hand, alcohol allows shorter ranges at low temperatures. Indeed, mercury thermometers can reach temperatures ranging from -30 to $+300$ ^oC. By using an inert gas

[1] $T_C = (T_F - 32)/1.8$.
[2] $T_F = 32 + 1.8T_C$.

(as nitrogen) in the capillary above the mercury, higher temperatures of up to 500 ^oC can be reached.
The working process is linked to the expansion of the liquid due to the temperature variation (much higher than that of the glass forming the bulb and capillary). The liquid rises in the capillary and the reached level indicates the temperature value. These thermometers can be fully or partially immersed; conversion formulas allow account to be taken of temperature conditions other than ideal ones.

1.1.2 Bimetallic Lamina Thermometer

Two sheets of different metals, with different thermal expansion coefficients, are connected together formin a lamina; when the lamina is subjected to temperature changes, it bends upwards or downwards. The relationship between the curvature radius, r, and the change in temperature is:

$$r = \frac{3m_1 + m_2}{6\alpha^* \; T^* \; m_1} \; t \tag{1.1}$$

with:

$$\begin{aligned} m_1 &= (1+m)^2 \\ m_2 &= (1+mn)(m^2 + 1/mn) \\ \alpha^* &= \alpha_2 - \alpha_1 \\ T^* &= T - T_0 \end{aligned}$$

where:

t	=	overall lamina thickness
m	=	ratio of thicknesses ($\alpha_{min}/\alpha_{magg}$)
n	=	ratio of elastic modules ($\alpha_{min}/\alpha_{magg}$)
α_1	=	lower thermal expansion coeff.
α_2	=	higher thermal expansion coeff.
T	=	measurement temperature
T_o	=	reference temperature (corresponding to the temperature at which the lamina was manufactured)

If, as is usually the case, $m \simeq 1$, $n \simeq 1$, $(n+1)/n \simeq 2$, then:

$$r \simeq \frac{2t}{3\alpha^* \; T^*} \tag{1.2}$$

Table 1 shows the thermal expansion coefficients and the elastic modules for some of the materials used in bimetallic lamina thermometers.

material	$\alpha * 10^{-6}\,^oC^{-1}$	$E\,(GPa)$
invar (Fe 64% – Ni 36%)	1.7	150
brass	20.2	100
monel 400 (Ni 67% – Cu 33%)	13.5	190
inconel 702	12.5	220
steel 316	16.0	200

Tab. 1 - Thermal dilatation coefficients and elastic modules of some materials used in bimetallic lamina thermometers

Since it is useful to have large α^*, materials with small α are the preferred materials of choice: tipically, *invar* is used. It is also possible to have negative α values (for example with carbon fibre composites).
For a thermometer consisting of an invar foil and a steel foil (both of thickness equal to 0.5 mm), with reference temperature $T = 20\ ^oC$ and measurement temperature $T = 115\ ^oC$, the radius of curvature can be calculated as follows: $t = 1\ mm$, $m = 1$, $n = 0.75$, $\alpha_1 = 1.7$, $\alpha_2 = 20.2$, $r = 381.29\ mm$.

1.1.3 Resistance Thermometer

Many materials can be used as temperature-sensitive resistive elements. The resistance-temperature relationship, at least over a limited temperature range, can be considered linear:

$$R(T) = R_0(1 + \gamma\Delta T) \tag{1.3}$$

where R_0 is the resistance at the reference temperature, γ is the coefficient of variation of resistance with temperature ($^oC^{-1}$) and ΔT is the variation of temperature from its reference value. If larger temperature variations are considered, the relationship can no longer be regarded as linear:

$$R(T) = R_0(1 + a\Delta T + b\Delta T^2) \tag{1.4}$$

where a and b are constants depending on the material. A focus on the "apparent temperature", i.e. the temperature due to causes other than changes in resistance such as the effect of stress, is needed.
In the case of a platinum resistance thermometer, linear behavior (within $\pm 0.4\%$) occours over

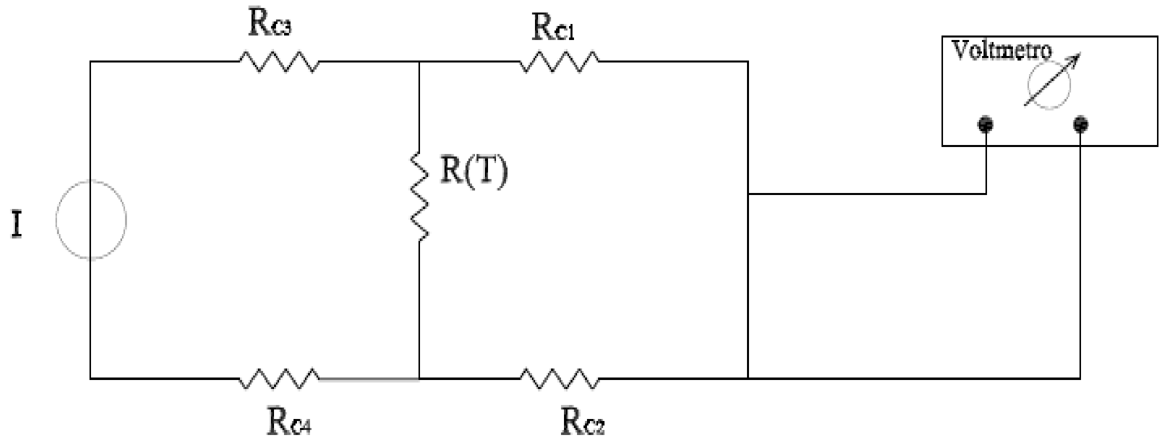

Figure 1.1: Electric scheme of a resistance thermometer.

a range of temperature from $-180\ ^oC$ to $150\ ^oC$. For example, with a resistance $R_0 = 100\ \Omega$, from Eq.1.3, the sensitivity is:

$$S = \frac{\partial R}{\partial T} = \gamma R_0 \tag{1.5}$$

which, considering $\gamma = 40 \times 10^{-4}\ ^oC^{-1}$ and $R_0 = 10\ \Omega$, leads to $S = 0.04\ \Omega/^oC$.
In the case of resistance thermometers, the measurement is carried out with a bridge circuit (in a similar way to strain measurements with resistance strain gauges); there are several problems concerning the resistance of the connecting cables and, in general, the measure of "apparent temperature" which are linked to resistance variations related to causes other than the temperature variation to be measured.
In addition to the classic bridge circuit, different measurement techniques can also be used. With a connection of the type shown in Fig.1.1, characterised by the use of a high-impedance voltmeter and a four-wire circuit, the advantage is that the effect of the connecting cables is negligible. Indeed, the current source provides a load-indipendent current which is not affected by the resistances R_{c_3} and R_{c_4}, while the voltmeter reading doesn't depend on the resistances R_{c_1} and R_{c_2}.
The resistance R_o can vary from 10 to 25000 Ω. The higher values reduce the effects of connecting cables and contact resistances.

1.1.4 Thermistors

Thermistors are semiconductors with, typically, negative temperature coefficients. The resistance varies exponentially with the temperature:

$$R(T) = R_0 e^{\beta\left(\frac{1}{T} - \frac{1}{T_0}\right)} \tag{1.6}$$

where R_0 is the resistance at the reference temperature, T and T_0 are respectively the working and reference temperature in K, β is a material- and temperature-dependent constant with a numerical value in the region of 4000 ^oK.
A resolution of aproximately .01 ^oC can be achieved. If a thermistor is used in an electric circuit, current flows through the element and heating effect occurs, raising the sensor temperature to an equilibrium condition. Thermistors are used to compensate for the effects of temperature variation in a circuit: their negative variation with temperature is used to offset the positive variation of other components in the circuit. The sensitivity can be obtained from Eq.1.6:

$$S = \frac{\partial R}{\partial T} = -\frac{\beta R_0}{T^2} e^{\beta\left(\frac{1}{T}-\frac{1}{T_0}\right)} \tag{1.7}$$

Evaluating the resistance coefficient:

$$\gamma^* = \frac{\partial R}{\partial T}\frac{1}{R} = \frac{-\beta}{T^2} \tag{1.8}$$

for $\beta = 4000$ ^oK and $T = 298$ ^oC, one gets $\gamma^* = -0.045$ (which is one order of magnitude higher than the resistance coefficient of platinum). At a temperature of 25 ^oC, the resistance can vary from 500 Ω to 10 $M\Omega$. The operating temperature can vary from -200 ^oC to 1000 ^oC (although this temperature range is not related to a single thermistor).
With silicon semiconductor and some boron impurities, one can obtain high value of temperature coefficients (both positive and negative). However, the resistance-temperature relationship is highly non-linear.

1.2 Thermocouple

Connecting two different metals, marked 1 and 2 in Fig. 1.2, produces a voltage between the ends A and B which depends on the temperature, T_1 and T_2, of the junctions (Seebeck effect). If the terminals A and B are closed on an external circuit, so that the current i flows, the terminal voltage changes (Peltier effect). In addition, if there is a temperature gradient on materials 1 or 2, the output voltage changes (Thomson effect).

These are three effects that influence the voltage in a thermoelectric circuit: the Seebeck effect, which links the output voltage, E_u, to the measuring junction temperature T_1 and to the reference junction temperature T_2, is the one of interest in the use of thermocouples as temperature transducers.
The presence of a third material in the measuring junction, Fig.1.3, does not alter E_u as long as the temperatures of the two junctions, T_1 and T_1', are the same. Furthermore, if the circuit between temperatures T_1 and T_2 (Fig.1.4) provides voltage E_1 and the same circuit between the temperatures T_2 and T_3 provides voltage E_2, then between T_1 and T_3 the circuit supplies voltage $E_3 = E_1 + E_2$.

The thermocouple circuits require two junctions. One of these, referred to as the "cold point", is at the reference temperature while the other is at the measurement temperature. The cold point may be melting ice placed in a Dewar vessel to obtain the zero degree centigrade reference. Standard thermocouple tables refer to a temperature of zero degrees centigrade.
The output voltage is given by a non-linear relation:

$$E_u = aT + bT^2 + cT^3 \tag{1.9}$$

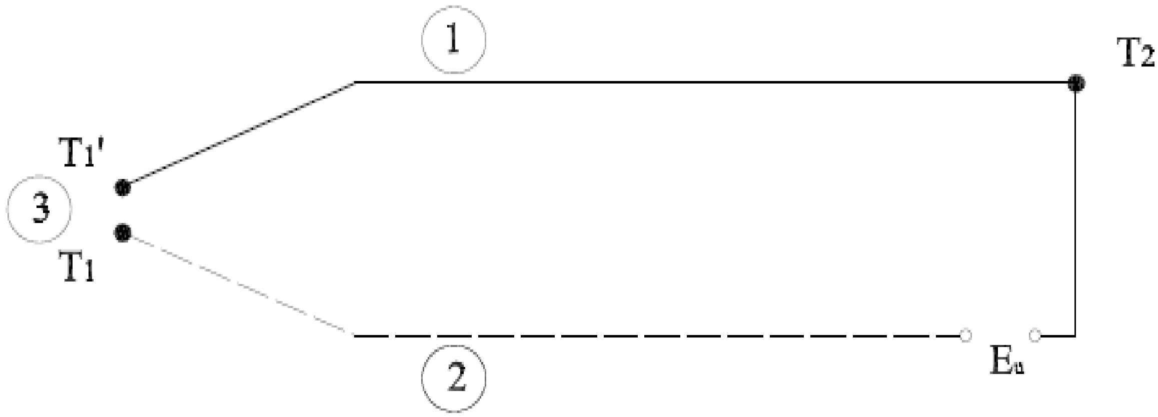

Figure 1.2: Elementary diagram of a thermocouple.

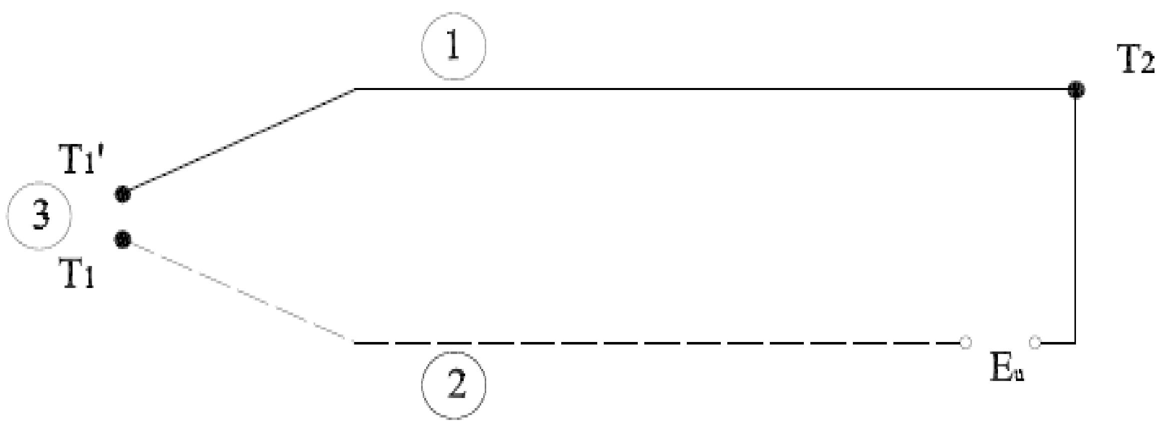

Figure 1.3: Effect of a third material on the thermocouple.

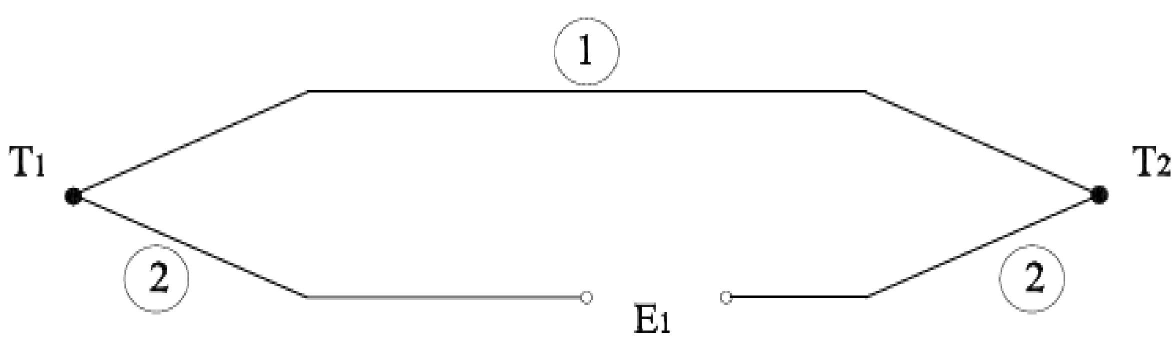

Figure 1.4: Scheme of the law of the intermediate temperature.

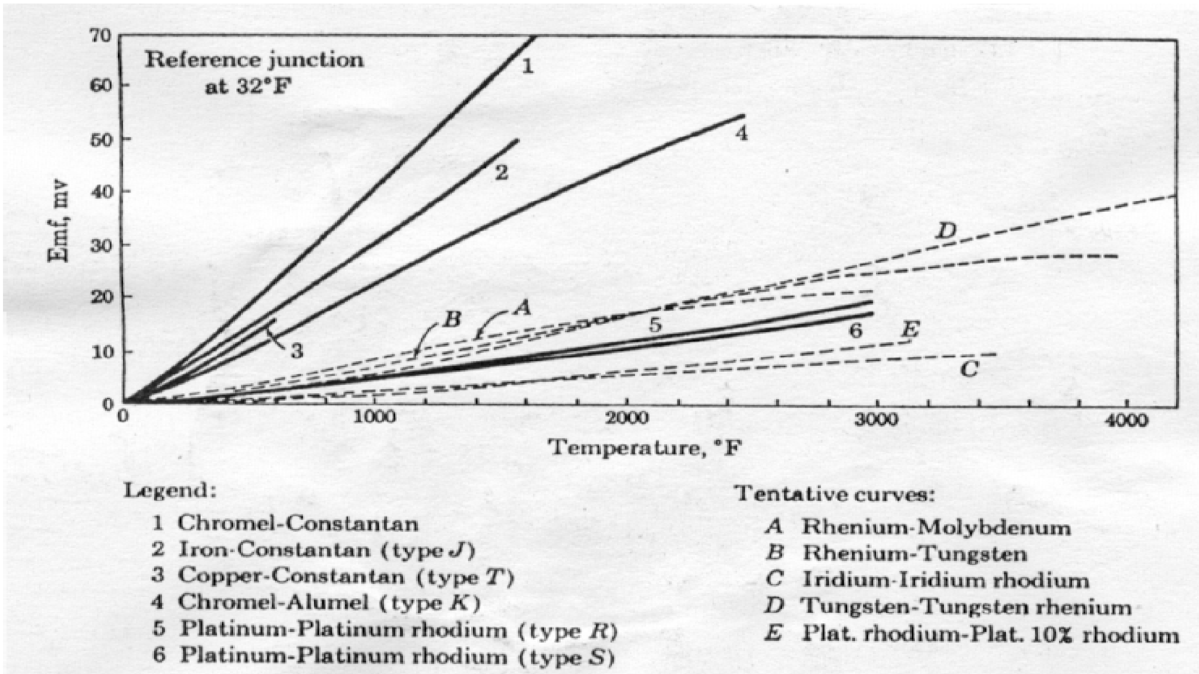

Figure 1.5: Characteristic curves of thermocouples.

where the constants a, b, c depend on the material. The sensitivity of a thermocouple is given by:

$$S = \frac{\partial E_u}{\partial T} = a + 2bT + 3cT^2 \tag{1.10}$$

Generally the sensitivity is of the order of some tens of $\mu V/^oC$ and its value is a function of temperature.
Tab.3 shows the abbreviations of different types of thermocouples, with their materials and field of application. The maximum sensitivity is given by T-type thermocouple at a temperature of 350 ^oC and is 60 $\mu V/^oC$; the minimum sensitivity is given by S-type thermocouple at a temperature of 100 ^oC and is 6 $\mu V/^oC$.
Fig. 1.5 shows the voltage trend as a function of temperature for several types of thermocouple. As an example, let us consider the case of an iron and constantan thermocouple with a reference temperature of $24\,^oC$. Table.2 gives: $E_{24} = 1.22\ mV$. If the output voltage, with reference temperature at $24\,^oC$, is $E_u = 3.59\ mV$, then the output voltage with reference temperature at $0\,^oC$ will be given by:

$$E_u(T) = 3.59 + 1.22 = 4.81mV \tag{1.11}$$

to which corresponds $T = 92\,^oC$ (as can be seen again from Tab.2).
To achieve a higher sensitivity, the thermocouples can be connected in series. In the case shown in Fig. 1.6, the three-junction thermopile provides an output voltage three times higher than the that of individual thermocouples (as long as the temperatures T_1 and T_2 are uniform).
A thermocouples connection in parallel, shown in fig. 1.7, can instead be used to obtain the

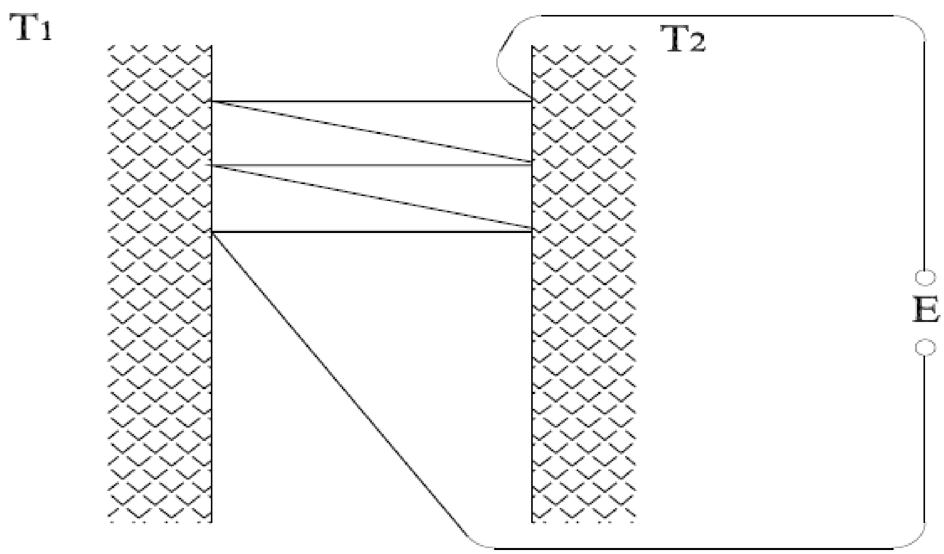

Figure 1.6: Thermocouples connected in series.

average value of the temperature at different measurement points. If the temperatures are different, each thermocouple provides a different value of voltage. But, with the parallel connection, the average value is collected.

Temp. $^\circ F$	Iron Costantana (J)	Copper Costantana (J)
-100	-3.49	-2.559
0	-0.89	-0.670
100	1.94	1.517
200	4.91	3.967
300	7.94	6.647
400	11.03	9.525

Tab. 2 - Table of thermocouples at ^oC

Of course, a thermocouple measures the temperature of the last point of electrical contact between the two materials making up the thermocouple. In the case of accidental connection between two cables of the thermocouple (Fig. 1.8), the measurement refers to the point of accidental contact and not to the point considered to be the point of measurement.

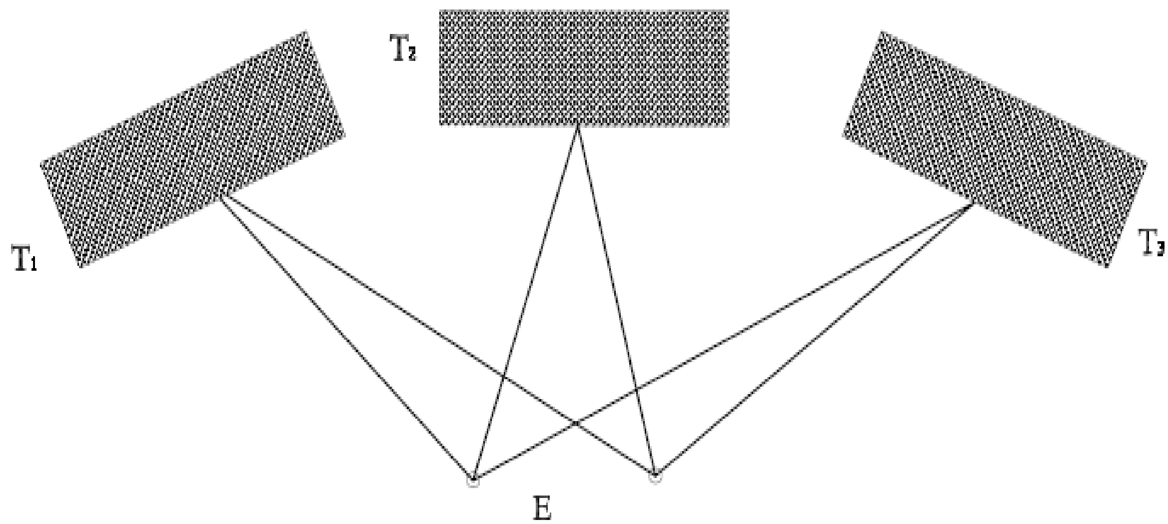

Figure 1.7: Parallel connection of thermocouples.

The standard materials used for thermocouples and their working temperature ranges are reported in Tab. 3.

type (material)	**range of temperature (^oC)**
B (Pt, Rh^+; Pt,Rh^-)	870—1700 ^oC
E ($Chromel^+$; $Costan^-$)	-180—870 ^oC
J (Fe^+; $Costan^-$)	-150—750 ^oC
K ($Chromel^+$; $Alumel^-$)	0—1260 ^oC
R (Pt^+; Rh, Pt^-)	0—1480 ^oC
S (Pt^+; Rh, Pt^-)	0—1480 ^oC
T (Cu^+; $Costan^-$)	-250—340 ^oC

Tab. 3 - Materials of thermocouples
(Chromel: Ni 90%, Cr 10%;
Alumel: Ni 94%, Mn 3%, Al 2%, Si 1%)

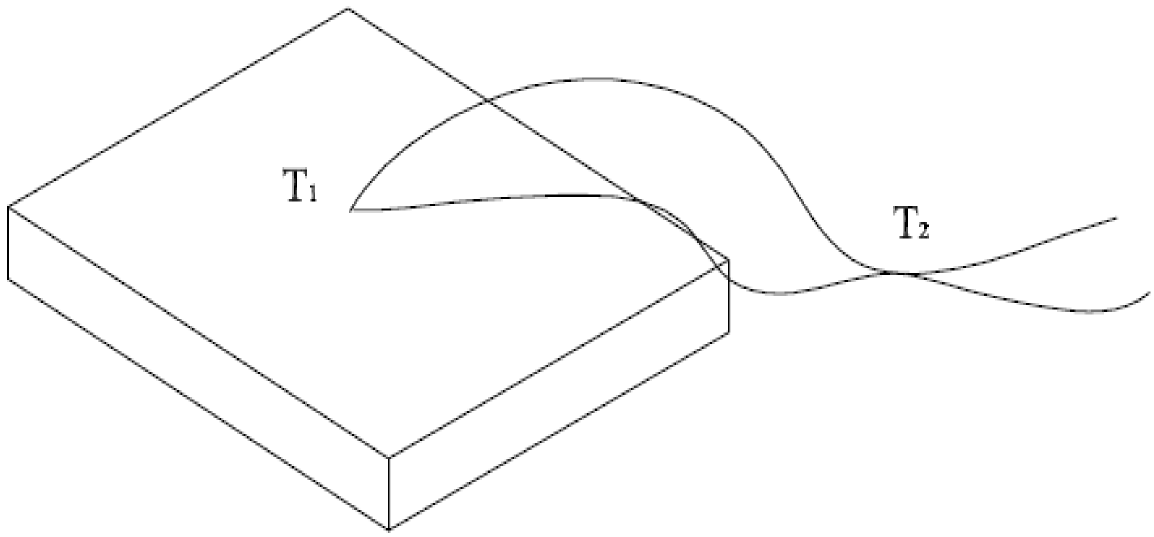

Figure 1.8: Accidental cable connection.

The measurement circuit consists of the measurement junction and the one connected to the clamps of the instrument which measures the output voltage (Fig. 1.9). By assuming that the clamps are at the same temperature and that their temperature is measurable, the measurement temperature can be evaluated from the thermocouple table.
For example, considering a thermocouple of T type (copper$^+$, costantana$^-$ - Fig. 1.9),if one read on the instrument $E_u = 2.877\ mV$ and the clamps temperature is about 24 ^oC, according to the intermediate temperature law, the correct output is:

$$E_{cu} = E_{u_{24}} + E_{24} \tag{1.12}$$

where E_{cu} is the correct output voltage with reference to the temperature of 0 ^oC, $E_{u_{24}}$ is the voltage measured with reference to the temperature of the clamps (24 ^oC for the examinated example) and E_{24} is the voltage of the cupper-costantana thermocouple at the temperature of 24 ^oC. From the table:

$$E_{cu} = 2.877 + 0.952 = 3.829\ mV \tag{1.13}$$

Again from the table, the temperature corresponding to E_{cu} (Eq.1.13) can be evaluated:

$$T = 90\ ^oC \tag{1.14}$$

The wire that makes up the thermocouple is considerably more expensive than normal electrical cable. Therefore, when it comes to measurements at a distance from the recording system, it is usually necessary to limit its length by using connecting cables (as shown in Fig. 1.10). Of course, it is necessary that the temperature T_2 of the connecting junctions (q_1, q_2) is uniform, i.e. the same at both joints, and accurately measured. The need to measure the temperature of the connecting junctions is, particularly for the industrial measurement, a negative feature:

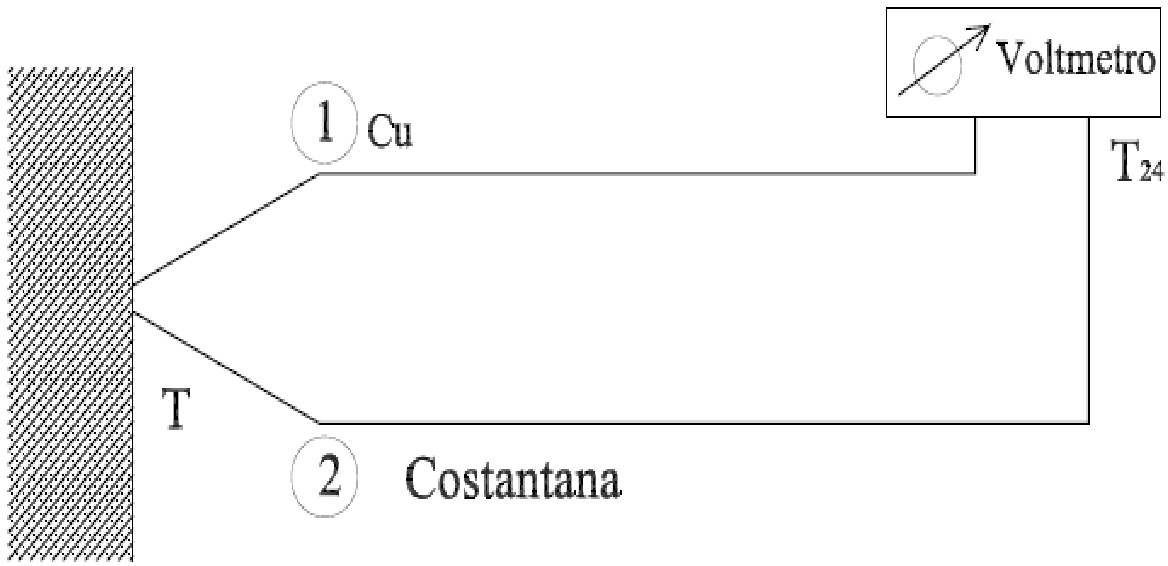

Figure 1.9: Reading correction with reference temperature.

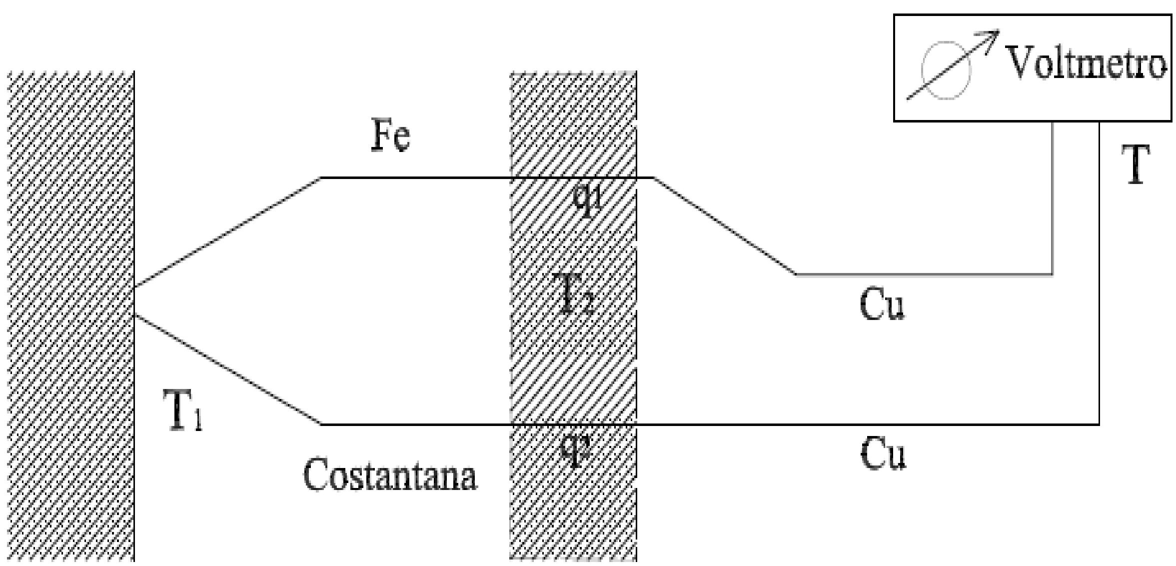

Figure 1.10: Scheme with connection cables and controlled temperature.

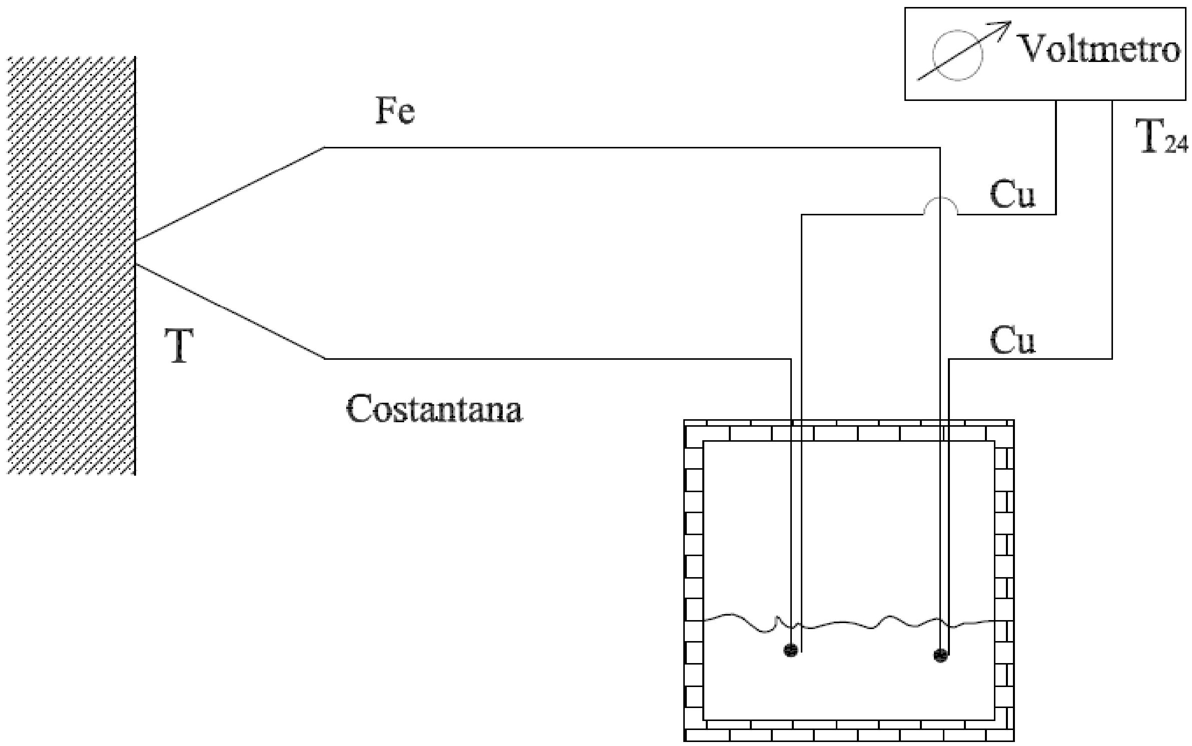

Figure 1.11: Thermocouple with compensation circuit.

it is possible to avoid this measurement by using compensation cables (which behave as if they were made of the same material as the thermocouple) instead of a normal conductor. The use of compensating cables is typical of industrial measurements, but it is not precise. For laboratory measurements, temperature-controlled reference junctions can be used, e.g. with a melting ice cockpit. For high-precision measurements, distilled water can be used in the cockpit to eliminate possible temperature variations due to contamination in the water (Fig. 1.10).
The output voltage of a thermocouple depends on its manufacturing specifications and on ageing. In the case of precision measurements, initial calibration is required, either by reference to standard measuring points or by comparison with calibrated sensors. Also periodic calibration checks are needed.
In Fig. 1.11, a widely used measurement scheme is shown. The thermocouple wires are connected to a block which is maintained at an uniform temperature, but which varies according to the room temperature. The value of this reference temperature is measured, e.g. with a resistance thermometer, and it is then processed by a compensation circuit, which provides a compensation voltage to be sent to the measuring voltmeter. In the case of a "multi-channel" instrumentation, several thermocouples (even of different types) are connected to the uniform temperature block and the compensation circuit processes the compensation voltage for each channel.
For measurements at very high temperatures, such as in the case of jet and rocket engines with a range of temperature from 1000 to 2500 ^oC, innovative thermocouples such as rhodium-irinium, tungsten-rhenium and boron-graphite have been developed.
Rhodium-iridium thermocouples can be used up to 2200 ^oC and they have a sensitivity around

$6\ \mu V/^oC$. Tungsten-rhodium thermocouples can work up to 2700 ^oC with similar sensitivity. Boron-graphite thermocouples can achieve higher sensitivities, up to 40 $\mu V/^oC$.

1.3 Measurement of temperature by frequency variations

A recently developed method of temperature measurement is based on the temperature sensitivity of the resonance frequency of a quartz crystal: a linear relationship between resonance frequency and temperature can be obtained with a temperature resolution of up to one thousandth of a degree Celsius. This relationship has been known for a long time, but it is only recently that an advantageous orientation of the crystal, referred to as "linear cut-off", has been identified, allowing a sensitivity of 1000 $Hz/^oC$ with a linearity of 0.05 % over a temperature range between $-40\ ^oC$ and 230 ^oC (this is a much higher linearity value than is typical, e.g. in the case of a platinum resistance thermometer the linearity is only 0.55 %).
The nominal resonance frequency is 28 MHz and the sensor output is compared with a frequency 28.208 MHz, provided by an oscillator; the frequency difference is detected, converted into pulses and sent to a counter. As the time constant is of the order of a second, extremely high resolutions can be achieved (even in the order of .0001 ^oC over a measurement time of 10 s).

1.4 Very high temperature measurements

High temperatures can be measured with appropriate thermocouples. Other temeprature measurement techniques of particular interest for very high temperatures are based on radiation measurements (e.g. temperature measurements in industrial oven). In the case of the optical pyrometer, the evaluation of the temperature of a surface is related to the colour of the radiation it emits.[3]
The term "pyrometry" denotes temperature measurements that refer to different types of thermal radiation. All bodies at a temperature above absolute zero, radiate energy and also receive and absorb it from other bodies. The intensity of emitted and absorbed radiation enegy depends on the temperature and properties of the bodies. Wave hitting the surface are absorbed, reflected and transmitted. By indicating with α the absorption coefficient, with ρ the reflection coefficient and with τ the transmission coefficient:

$$\alpha + \rho + \tau = 1 \tag{1.16}$$

In case of an ideal "reflector" (condition approached by a body with mirror-like surfacing), $\rho \to 1$; in case of an ideal "transmitter" (as some gases), $\tau \to 1$; in case of an ideal "absorber" (i.e. a black body), $\alpha \to 1$. If one distinguishes emission from absorbtion, an emissivity coefficient

[3] For the radiated power from a source in a certain direction, one generally has an expression like (Plank):

$$L_\lambda = \frac{c_1}{\lambda^5 \left(e^{\frac{c_2}{\lambda T}} - 1 \right)} \tag{1.15}$$

which is expressed in watts per square meter per steradian. The constants c_1 and c_2 depend on the characteristics of the source. The function has a relative maximum for a wavelength $\lambda_{max} = \alpha/T$ ($\alpha = cost$), which provides the dominant component of the light spectrum (for example the visible one). This wavelength can be directly correlated with temperature.

must also be defined. This coefficient is denoted by ϵ and results to be $\epsilon = \alpha$. Thus, an ideal "absorber" is also an ideal radiator The energy exchange between two ideal radiators, indicated with A and B, is given by the Stefan-Boltzmann's law (with $\epsilon = 1$):

$$q = \sigma(T_A^4 - T_B^4) \tag{1.17}$$

Considering no-ideal characteristics of the bodies:

$$q = \sigma\epsilon C_{AB}(T_A^4 - T_B^4) \tag{1.18}$$

where q is the heat flow (expressed in W/m^2), C_{AB} is a geometrical factor that depends on the position and geometry of the bodies A and B, T_A and T_B are the absolute temperatures of A and B, σ is the Stefan-Boltzmann constant (with value $5.729\,10^{-8}\,W/m^2K^4$). This relationship forms the basis of total radiation pyrometry. From a practical point of view, however, there are several problems requiring different calibration procedures (such as geometry and position of the bodies and absorption of the medium). The total radiation absorbed by the pyrometer is measured with temperature sensors (usually thermocouples arranged in series to increase sensitivity).