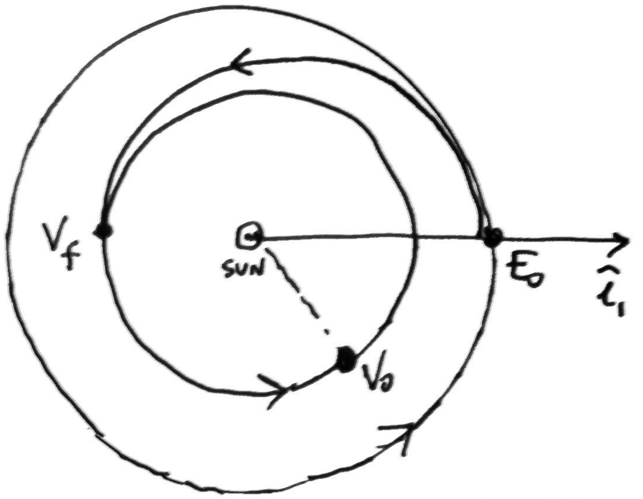
Classwork example – solution –

(1)

(a) $\Delta t = \frac{T}{2} = \pi\sqrt{\frac{a_H^3}{\mu_S}}$



$$a_H = \frac{r_V + r_E}{2} = 128\,850\,000 \text{ km}$$

$$e_H = \frac{r_E - r_V}{r_E + r_V} = 0.160$$

$\rightarrow \Delta t = 12613658$ sec

$= 146.99$ days

$$\rightarrow \phi_{vf} = \pi = \phi_{v0} + \sqrt{\frac{\mu_S}{r_V^3}}\,\Delta t$$

Angle of Venus at t_f — Angle of Venus at t_0

$E_{0/f}$ = Earth at initial (0) time and final (f) time

$V_{0/f}$ = Venus at initial (0) time and final (f) time

$\phi_{v/c}$ = angle taken counterclockwise from $\hat{i}_1$

$$\rightarrow \phi_{v0} = \pi\left(1 - \sqrt{\frac{a_H^3}{r_V^3}}\right) = -0.941 = -53.9 \text{ deg}$$

(b) $v_{r,f}^{(H)} = 0$ $\qquad v_{\theta,f}^{(H)} = \sqrt{\frac{\mu_S}{p_H}}\,(1+e_H) = 37.722\ \frac{\text{km}}{\text{sec}}$

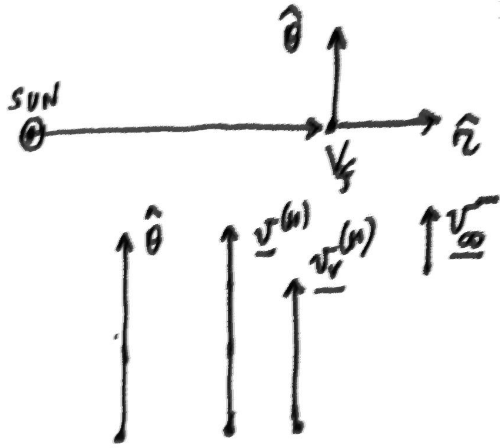
$p_H = a_H(1-e_H^2)$

$$\underline{v}_\infty^- = \underline{v}^{(H)} - \underline{v}_V^{(H)} \longrightarrow \underline{v}_\infty^- = v_\infty\,\hat{\theta} = \left(v_{\theta,f}^{(H)} - \sqrt{\frac{\mu_S}{r_V}}\right)\hat{\theta}$$

Helioc. velocity of spacecraft — Helioc. velocity of Venus

$= 2.702\ \frac{\text{km}}{\text{sec}}$

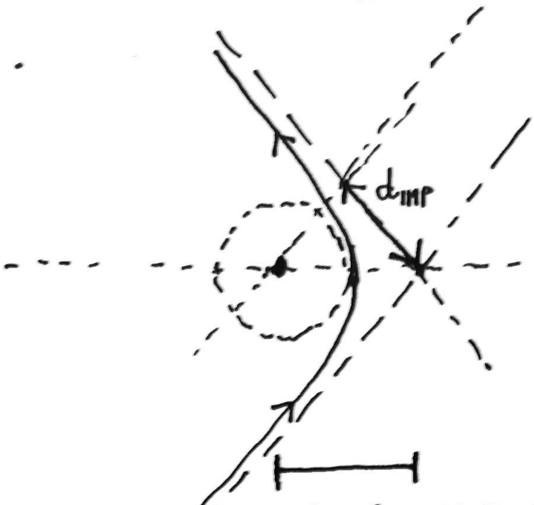
From v_∞ one obtains



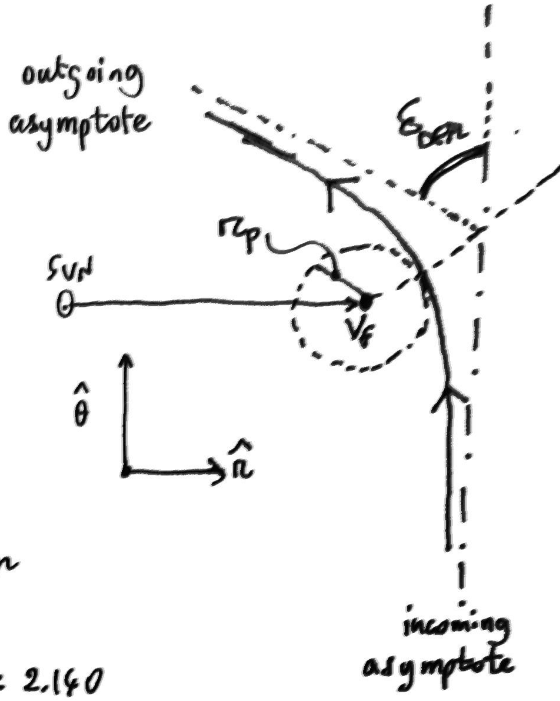
$$a_{HYP} = -\frac{v_\infty^2}{\mu_V} = -44611 \text{ km}$$

(c) $r_p = a_{HYP}(1 - e_{HYP}) = R_V + 200 \text{ Km}$

$$\rightarrow e_{HYP} = 1 - \frac{r_p}{a_{HYP}} = 1.140$$



$-a_{HYP}\, e_{HYP} = 50863 \text{ Km}$



$$\delta_{DEFL} = 2\,\theta_*^{lim} - \pi = 122.6 \text{ deg} = 2.140$$

$$d_{IMP} = -a_{HYP}\sqrt{e_{HYP}^2 - 1} = 24432 \text{ Km}$$

(d) After flyby

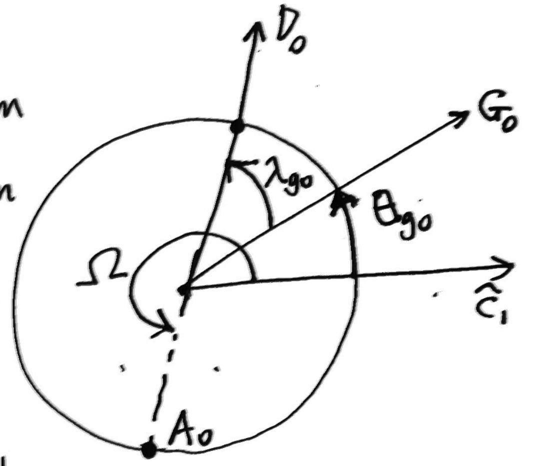
$$\underline{v}^{(H)} = \underline{v}_\infty^+ + \underline{v}_V^{(H)} \quad \text{where} \quad \begin{cases} v_{\infty,r}^+ = -v_\infty \sin \delta_{DEFL} = -2.277 \frac{\text{km}}{\text{sec}} \\ v_{\infty,\theta}^+ = v_\infty \cos \delta_{DEFL} = -1.455 \frac{\text{km}}{\text{sec}} \end{cases}$$

In the end, after flyby the spacecraft has the following components of heliocentric velocity:

$$\begin{cases} v_r = v^+_{\infty,r} + v^{(H)}_{V,r} = -2.777 \ \frac{km}{sec} \\ v_\theta = v^+_{\infty,\theta} + v^{(H)}_{V,\theta} = 33.565 \ \frac{km}{sec} \end{cases}$$

and the velocity magnitude is $v_+ = 33.642 \ \frac{km}{sec}$ ($< 37.722 \ \frac{km}{sec}$, velocity prior to flyby).

(2) (a) $T = 2\pi\sqrt{\frac{a^3}{\mu}} \rightarrow a = 7136.635$ km $= R_E + 758.5$ Km



Due to J_2, the sunsynchronism condition allows finding i:

$R = a$ (circular orbit)

$$i = a\cos\left[-\frac{2\pi}{T_{1yr}}\frac{2a^{7/2}(1-e^2)^2}{3R_E^2 J_2\sqrt{\mu_E}}\right] = 1.718 = 98.5 \text{ deg}$$

$e=0$, $T_{1yr} = 86164 \cdot 365.25$ sec

G_0 = Greenwich meridian at t_0

D_0 = descending node at t_0

A_0 = ascending node at t_0

($\lambda_{g0} = 15$ deg, $\theta_{g0} = 45$ deg)

From the figure $\Omega = \theta_{g0} + \lambda_{g0} + \pi = 240$ deg

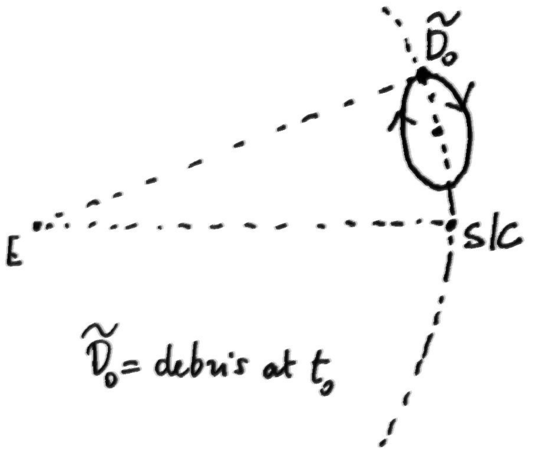
(b) Ascending node is flown after half period, i.e.:

$\Delta t = 50$ min

$\theta_{gf} = \theta_{g0} + \omega_E \Delta t$, then $\lambda_{gf} = \Omega - \theta_{gf} = -3.099 = -177.5$ deg

(c) $x_0 = 0$ km $\quad y_0 = 8$ km

$\dot{x}_0 = 2 \frac{m}{sec} \quad \dot{y}_0 = 0 \frac{m}{sec}$



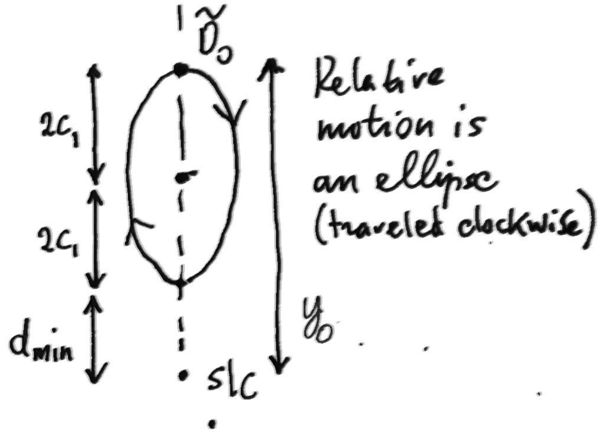
$$\begin{cases} x(t) = \dfrac{\sin(w_R t)}{w_R} \dot{x}_0 \\ y(t) = y_0 + \dfrac{2c_1}{w_R}\left[\cos(w_R t) - 1\right] \end{cases}$$

in this case, where

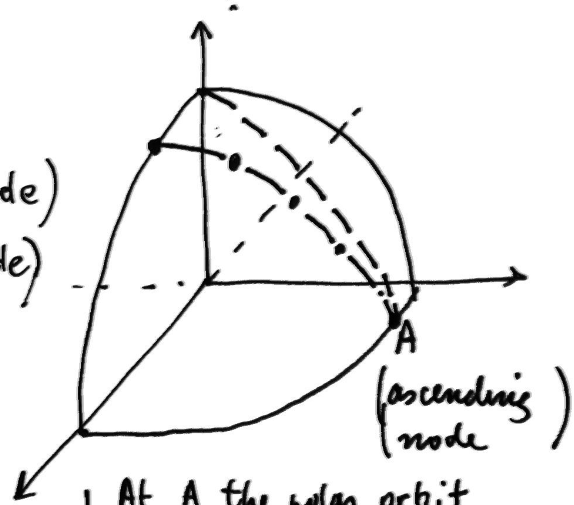
$$c_1 = \frac{|\dot{x}_0|}{w_R} = \frac{2 \cdot 10^{-3}\ \text{Km}}{2\pi} T =$$

$= 1.910$ Km

$\rightarrow d_{min} = y_0 - 4c_1 = 361$ m



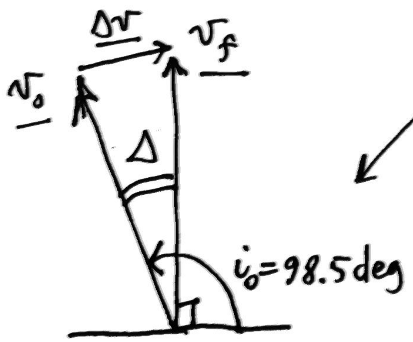
(d) Polar orbit is obtained by a single velocity change at either A (ascending node) or D (descending node)



$\downarrow$

$|\underline{\Delta v}| = 2v_0 \sin\frac{\Delta}{2}$

$= 1.102 \frac{\text{Km}}{\text{sec}}$



$\Delta = |i_0 - i_f|$

$= 8.5$ deg

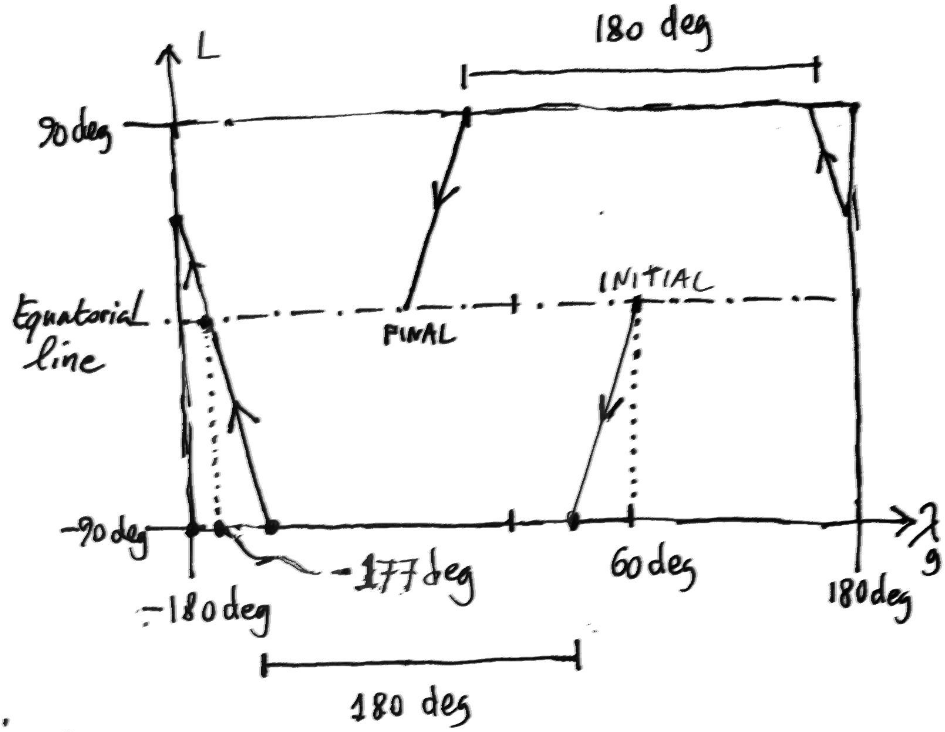
At A the polar orbit (curve ⋱) intersects the sunsynchronous orbit (curve ⋮)

The same occurs at the descending node (not shown)

(e)

Initial geo long was found at point (a)

Geo long at ascending node was found at point (b)



Ground track of polar circular orbit includes straight lines traveled westward

180-deg jumps occur at polar latitude $\left(L = \pm \frac{\pi}{2}\right)$

The figure depicts the ground track in 1 orbital period.

(f) Given $\rho_{800} = \rho(800 \text{ km})$ and $\rho_{700} = \rho(700 \text{ km})$

$$\rho = \rho_0 \exp\left[-\frac{H - H_0}{H_{sc}}\right] \rightarrow \quad H_{sc} = -\frac{H_f - H_0}{\ln \frac{\rho_f}{\rho_0}} = 88.7 \text{ km}$$

$\rho_0 = \rho_{700}$, $\rho_f = \rho_{800}$

$H_f = 800$ km, $H_0 = 700$ km

$$\rho_s = \rho_0 \exp\left[-\frac{H_s - H_0}{H_{sc}}\right]$$

$$= 1.601 \cdot 10^{-5} \frac{\text{kg}}{\text{km}^3}$$

$$\rightarrow a_{fin} = \left[a_{ini}^{1/2} - \frac{C_D S}{m} \rho \sqrt{\mu_E}\, \Delta t\right] = 7135.860 \text{ km}$$

$\Delta t = 90 \cdot 86400$ sec

$\rightarrow \Delta r = 773$ m (decrease in altitude)

(3) See solution of exercise 3, set 7