Spaceflight Mechanics

15 December 2022 – CLASSWORK EXAMPLE – part 2

Orbital Mechanics

1. An interplanetary mission starting at Earth and headed to Venus is planned. Assuming that the interplanetary flight is approximated by a Hohmann transfer, evaluate

(a) transfer duration and phase angle between the planets at departure,

(b) the heliocentric velocity of the probe at Venus and the corresponding excess of hyperbolic velocity relative to Venus,

The spacecraft performs a direct (i.e. prograde, counterclockwise) flyby at Venus, with periapse altitude (over Venus) equal to 200 km. Calculate

(c) impact parameter and deflection angle of the planetocentric hyperbola,

(d) radial and horizontal components of the heliocentric velocity after the flyby.

Provide a schematic graphical representation of the trajectory.

	Gravitational parameter, μ [km^3/sec^2]	Orbit radius, R [km]	Radius of Planet l [km]
Sun	1.327e11	/	/
Earth	398600.4	149.5*10^6	6378.136
Venus	325700.0	108.2*10^6	6052

2. A satellite is placed in a circular sunsynchronous orbit about Earth, with orbital period of 100 minutes. At the initial time, it is at the descending node, at geographical longitude of 15 deg, and the Greenwich sidereal time equals 45 deg. The spacecraft has ballistic coefficient equal to 0.05 $\mathrm{m^2/kg}$.

(a) Calculate the orbit radius, inclination, and RAAN.

(b) Obtain the geographical longitude flown at the ascending node.

At the initial time, a nearby space debris is located 8 km ahead of the spacecraft (along-track displacement), has no radial displacement, has radial velocity of 2 m/sec, and no along-track relative velocity.

(c) Depict the relative trajectory of the debris with respect to the spacecraft, and calculate the distance of closest approach.

A single impulsive orbit maneuver is designed, to obtain a polar orbit.

(d) Identify the point(s) where the impulsive maneuver can be performed, and evaluate the $\Delta \boldsymbol{v}$ (magnitude and direction).

(e) Portray an illustrative sketch of the ground track of the polar orbit.

(f) Evaluate the orbit decay in 3 months, using the following two data for the atmospheric density: $\rho = 3.614 \cdot 10^{-14}\ \mathrm{kg/m^3}$ at altitude of 700 km, $\rho = 1.170 \cdot 10^{-14}\ \mathrm{kg/m^3}$ at altitude of 800 km.

Fundamentals of attitude dynamics

3. A cylindrical axisymmetric spacecraft has radius of 1 m, moments of inertia

$$I_1 = I_2 = I_T = 20 \text{ kg} \cdot \text{m}^2 \qquad I_3 = 35 \text{ kg} \cdot \text{m}^2$$

and is subject to no external torque. Axis $\hat{E}_3$ of the inertial reference frame is aligned with the angular momentum $\boldsymbol{H}_C\left(t_0^-\right)$; the spacecraft rotates with a constant nutation angle θ of 60 deg and a transverse velocity component $\omega_{12} = \sqrt{\omega_1^2 + \omega_2^2} = 0.3 \text{ sec}^{-1}$. At t_0^- $\omega_1\left(t_0^-\right) = -\omega_{12}$.

(a) Determine the angular velocity component ω_3.

(b) The spacecraft is equipped with two thrusters (see figure below), which are ignited for 1 sec at t_0 and provide a propulsive thrust $\boldsymbol{F}$, whose magnitude F equals 1 newton. Under the impulsive torque assumption, obtain the components (along the body axes) of the angular momentum $\boldsymbol{H}_C\left(t_0^+\right)$ after ignition of the two thrusters.

(c) A new inertial reference frame is defined, with axis $\hat{E}_3'$ aligned with $\boldsymbol{H}_C\left(t_0^+\right)$. Calculate the nutation angle at t_0^+, with respect to $\hat{E}_3'$.

(d) In the new inertial reference frame, the precession angle at t_0^+ is $\psi\left(t_0^+\right) = 0$. Determine the principal axis and angle associated with the instantaneous orientation of the spacecraft at time $t = t_0 + 60 \text{ sec}$.