KEPLERIAN TRAJECTORIES: EXERCISE SET 1

① Given the apoapse and periapse radii

$r_A = 30000\ km$ $\quad r_p = 8000\ km$

(a) Find a, e, p

(b) Find v_A, v_p, and the point (and value) at which the radial velocity is maximal

② Given the periapse radius and eccentricity of a hyperbola

$r_p = 7000\ km$ $\quad e = 3$

(a) Find a, p

(b) Find v_p and v_∞ (i.e. v as $r \longrightarrow \infty$) and $v_{r_{max}}$

③ Given the periapse radius along a parabola

$r_p = 7000\ km$

(a) Find p

(b) Find v_p and $v_{r_{max}}$

Solutions

① (a) $e = 0.579$ $\quad a = 19000\ km$ $\quad p = 12631.6\ km$

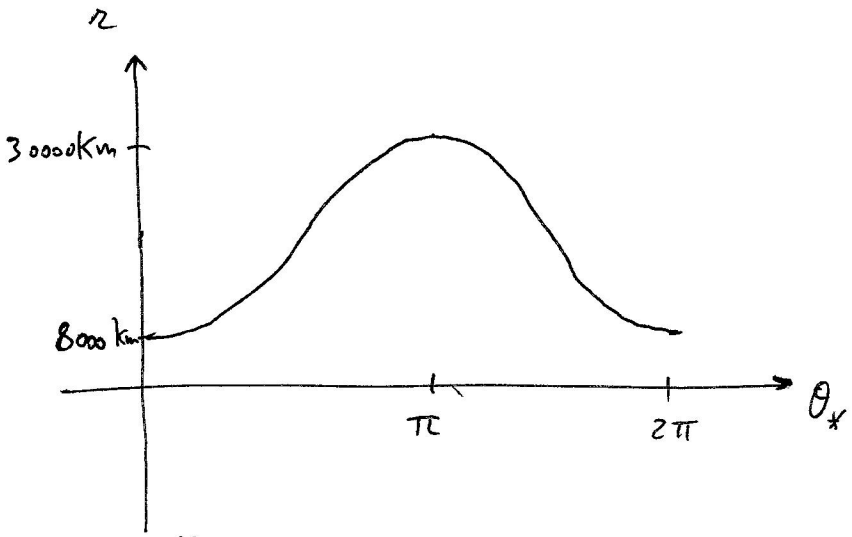
(b) $v_p = 8.870\ km/sec$ $\quad v_A = 2.365\ km/sec$ $\quad v_{r_{max}} = 3.252 \frac{km}{sec} \left(\theta_A = \frac{\pi}{2}\right)$

② (a) $a = -3500\ km$ $\quad p = 28000\ km$

(b) $v_p = 15.092 \frac{km}{sec}$ $\quad v_\infty = 10.672 \frac{km}{sec}$ $\quad v_{r_{max}} = 11.319 \frac{km}{sec}$

③ (a) $p = 14000\ km$ $\quad$ (b) $v_p = 10.672 \frac{km}{sec}$ $\quad v_{r_{max}} = 5.336 \frac{km}{sec}$

(1)

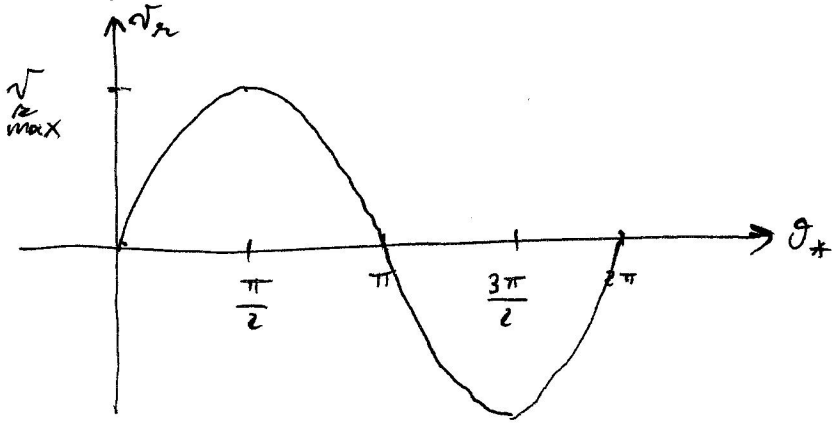


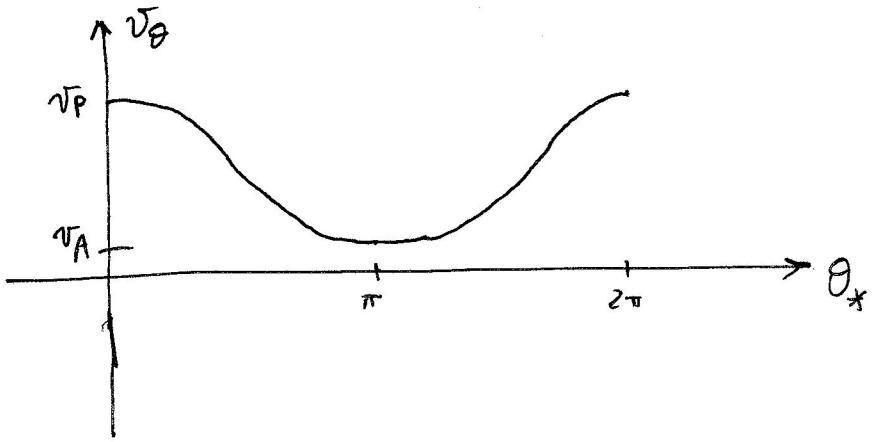
Note

$|\underline{r} \times \underline{v}| = r v_\theta = \text{const}$

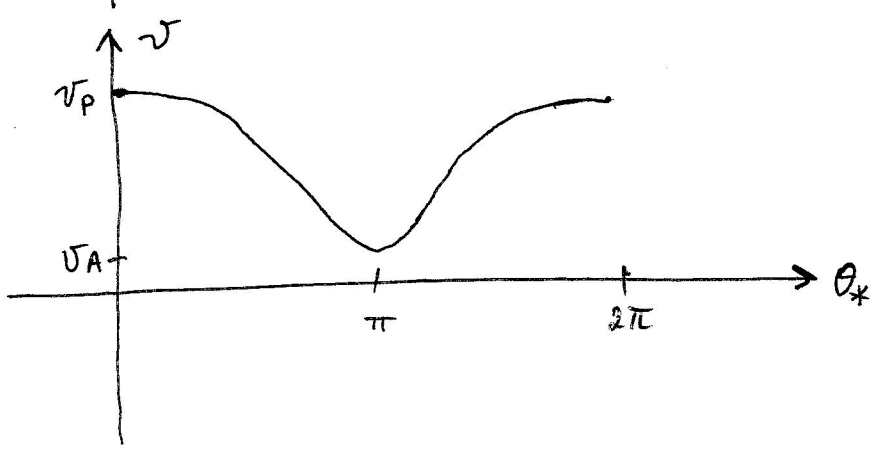
$r_A = 30000$ km

$r_p = 8000$ km



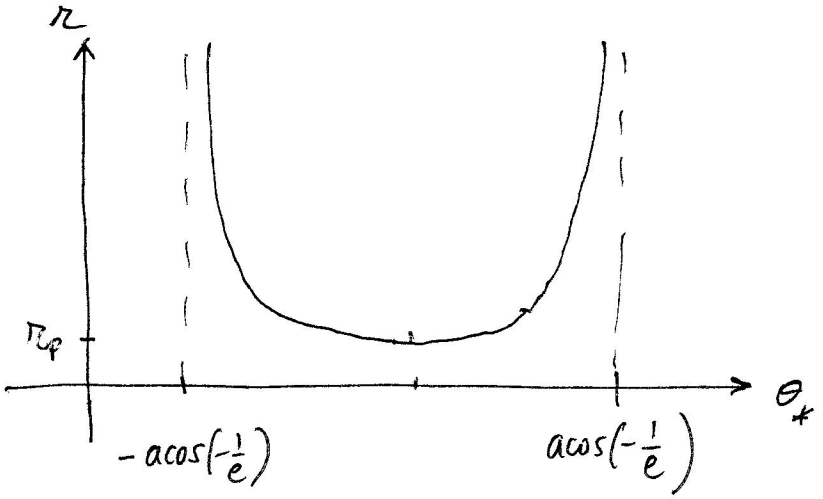


v_p = velocity at periapse



v_A = velocity at apoapse

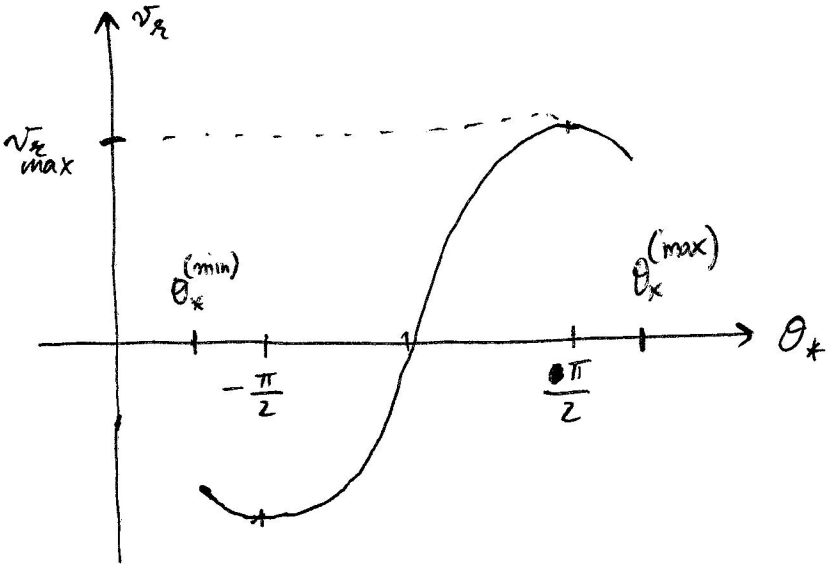
(2)



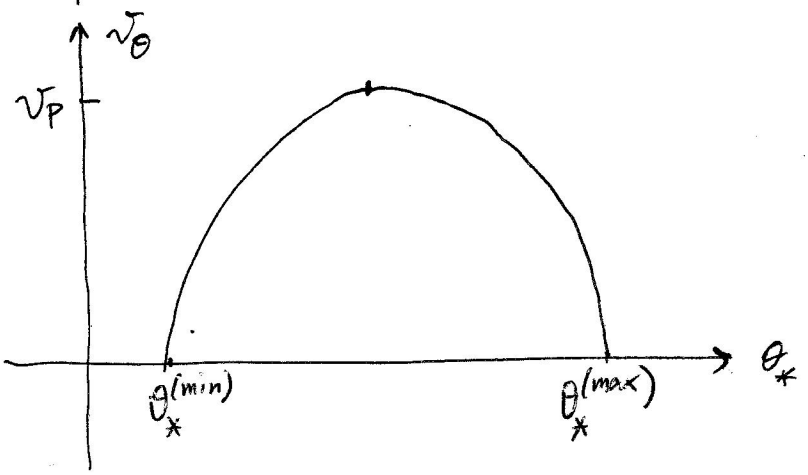
$\theta_*^{(max)} = \text{acos}(-\frac{1}{e})$

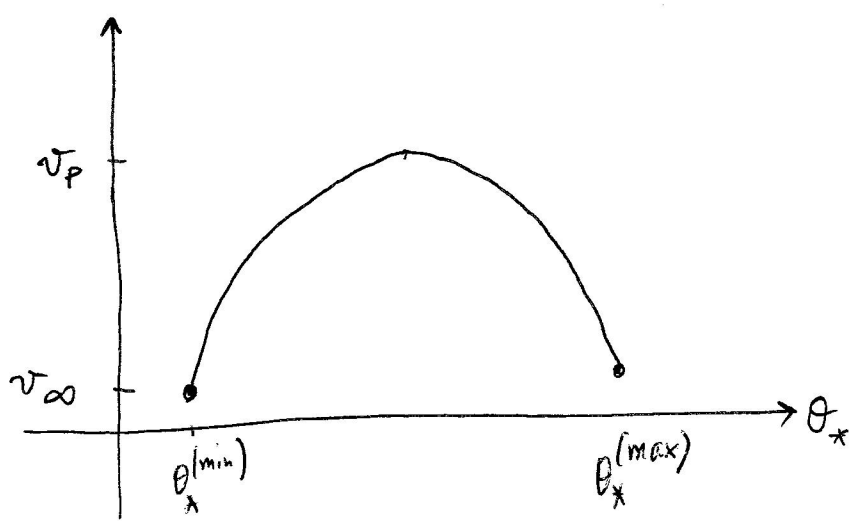
$\theta_*^{(min)} = -\text{acos}(-\frac{1}{e})$

r_p = radius at periapse



v_p = velocity at periapse

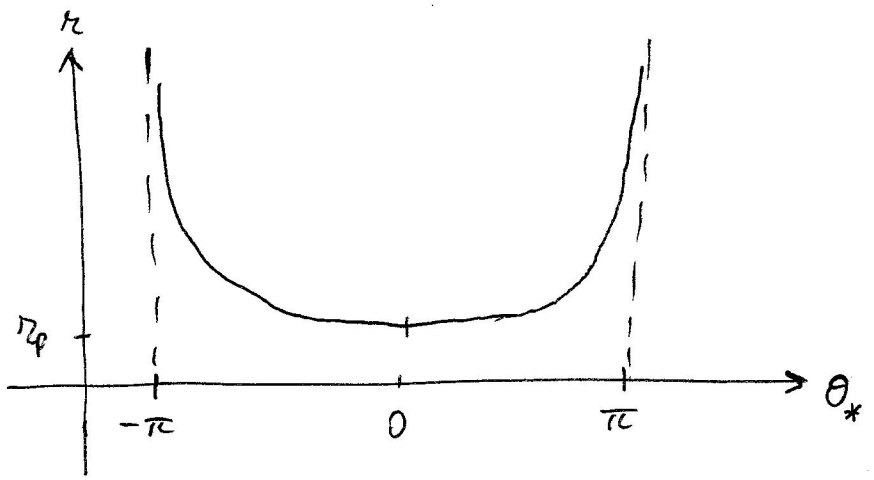




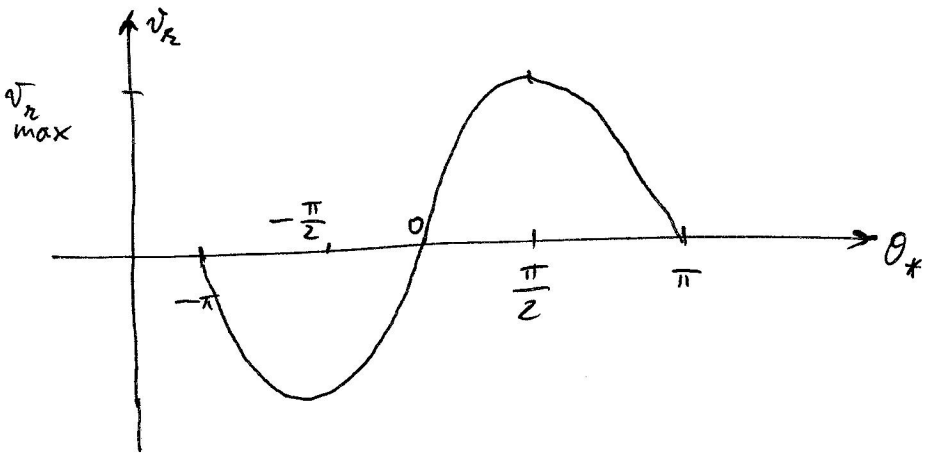
$v_\infty > 0$

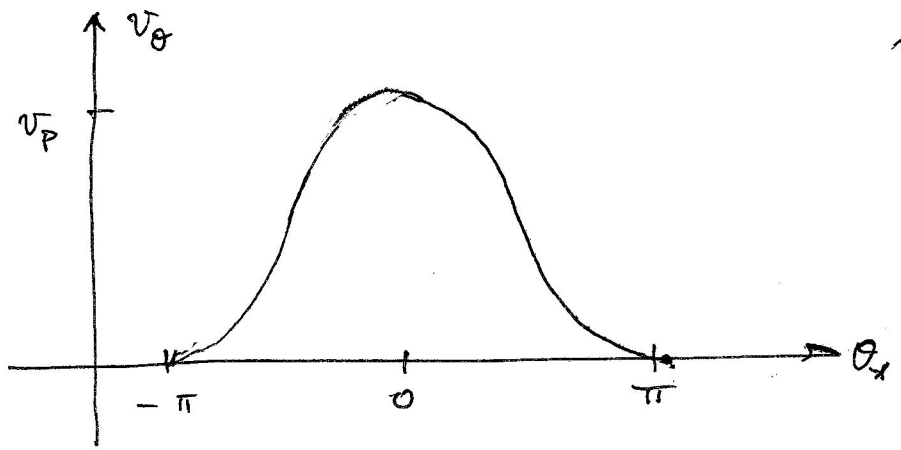
velocity at infinite distance (as $r \to \infty$)

(3)

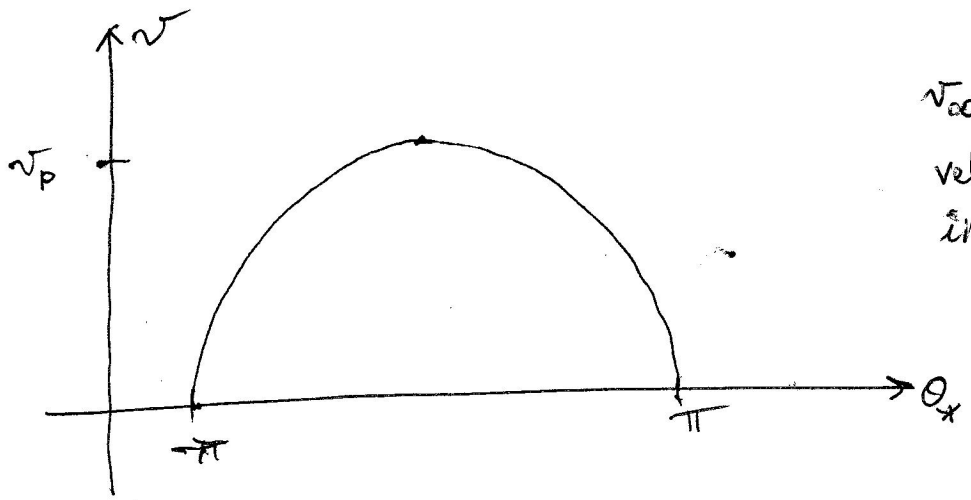


r_p = radius at periapse





v_p = velocity at periapse



$v_\infty = 0$
velocity at infinite distance
(as $r \to \infty$)