Spaceflight Mechanics

Exercise set 2

1. A spacecraft is placed on an Earth orbit with semimajor axis equal to $4R_E$ and perigee radius equal to $1.5R_E$ (where R_E denotes the Earth radius).

a. Obtain the true anomaly 4 hours after the perigee pass.

b. Calculate the corresponding radius *r* and the two velocity components (radial and horizontal).

Constants. $R_E = 6378.136$ km ; $\mu_E = 398600.4\ \text{km}^3/\text{sec}^2$ (Earth gravitational parameter)

2. An Earth orbit has perigee and apogee radii , r_P e r_A, and initial mean anomaly M_0 equal to

$$r_P = R_E + 400\ \text{km}, \quad r_A = R_E + 20000\ \text{km}, \quad M_0 = 30\ \text{deg}$$

a. Obtain the semimajor axis *a*, the eccentricity *e*, the semilatus rectum (parameter) *p*, the magnitude of the (specific) angular momentum *h*, and the (specific) energy Σ ;

b. Plot the time histories of the radius $r(t)$, velocity magnitude $v(t)$, and the two components $v_r(t)$ e $v_\theta(t)$ in an orbital period;

c. Interpret the time histories of $r(t)$ e $v(t)$ according to the 2nd Kepler's law

d. Calculate the maximum value of the flight path angle and the true anomaly at which it occurs.

3. A ballistic missile is launched from the Earth surface with an initial velocity of 7.2 km/sec. It impacts the surface 10000 km far away from the launch site. The Earth atmosphere and other perturbations are neglected.

a. Prove that two ballistic arcs exist, associated with two different orbit eccentricities.

b. Calculate semimajor axis and eccentricity in these two cases.

c. Calculate the maximum altitude for both cases.

d. Calculate the time of flight in both cases.

e. Obtain the flight path angle at launch in both cases.

4. For an elliptic orbit (with arbitrary values of the semimajor axis and eccentricity)

a. obtain the time needed to travel from the periapse to the point where the orbit crosses the semiminor axis;

b. check if the result is correct in the special case corresponding to $e \to 0$

5. At a given time t_0 the dynamic state of an orbiting satellite corresponds to the following values of radius *r*, absolute longitude λ_a, latitude ϕ, flight path angle γ, velocity *v*, and heading angle ζ:

$$r(t_0) = R_E + 5000 \text{ km} \quad \lambda_a(t_0) = -110 \text{ deg} \quad \phi(t_0) = -20 \text{ deg}$$
$$\gamma(t_0) = 30 \text{ deg} \quad v(t_0) = 7.2 \text{ km/sec} \quad \zeta(t_0) = 150 \text{ deg}$$

a. Obtain the five constant orbit elements *a*, *e*, *i*, Ω, and ω.

b. Calculate the true anomaly at t_0, $\theta_*(t_0)$, and the corresponding eccentric anomaly E_0 and mean anomaly M_0.

c. Calculate the three Cartesian components of the position vector and the velocity vector.

6. At a given time t_0 the dynamic state of an orbiting satellite is associated with the following Cartesian components of the position and velocity vectors:

$$X(t_0) = 7000 \text{ km} \quad Y(t_0) = 7000 \text{ km} \quad Z(t_0) = 800 \text{ km}$$
$$V_x(t_0) = -4.1 \text{ km/sec} \quad V_y(t_0) = 4.6 \text{ km/sec} \quad V_z(t_0) = 0.6 \text{ km/sec}$$

a. Obtain the five constant orbit elements *a*, *e*, *i*, Ω, and ω.

b. Calculate the true anomaly at t_0, $\theta_*(t_0)$, and the corresponding eccentric anomaly E_0 and mean anomaly M_0.

c. Calculate the values of radius *r*, absolute longitude λ_a, latitude ϕ, flight path angle γ, velocity *v*, and heading angle ζ at t_0.