Spaceflight Mechanics

Exercise set 3

1. A spacecraft is placed on a circular orbit of radius R_1. A horizontal velocity variation occurs, with magnitude equal to 20% of the initial velocity $\left(v_{\theta,+} = 1.2 v_{\theta,-};\ v_{r,-} = 0,\ v_{\theta,-} = \sqrt{\frac{\mu}{R_1}}\right)$. After this impulse, the orbit is elliptic. Calculate

a. the true anomaly right after the velocity change;

b. the orbit eccentricity.

2. Repeat the preceding exercise by assuming a radial velocity change, i.e. $v_{r,+} = v_{r,-} + 0.2 v_{\theta,-}\ \left(v_{r,-} = 0\right)$.

3. A spacecraft is placed in a circular orbit, and a *horizontal* velocity change is applied, such that $v_{\theta,+} = b v_{\theta,-}$. Determine the value of *b* such that the trajectory after the velocity impulse is a parabola.

4. A spacecraft is placed in a circular orbit, and a *radial* velocity change is applied, such that $v_{r,+} = v_{r,-} + b v_{\theta,-}$. Determine the value of *b* such that the trajectory after the velocity impulse is a parabola.

5. Calculate the two velocity changes Δv_1 and Δv_2 required to perform a Hohmann transfer from an initial circular orbit of radius R_1 to a final circular orbit of radius R_2 ($R_1 = 7000$ km and $R_2 = 14000$ km).

6. A spacecraft must complete an orbit transfer between its initial circular orbit, of radius $R_1 = R_E + 500$ km, and a final circular orbit of radius $R_2 = R_E + 200000$ km $\left(R_E = 6378.136 \text{ km},\ \mu = 398600.4 \text{ km}^3/\text{sec}^2\right)$. Obtain

a. the total velocity change for a Hohmann transfer;

b. the total velocity change for a bielliptic transfer, with $r_{Ai} = R_E + 500000$ km (apoapse radius of the intermediate elliptic arcs);

c. the total velocity change for a biparabolic transfer;

d. the time of flight to complete these three transfers.

7. A satellite orbits the Earth, with a perigee altitude of 700 km and eccentricity 0.3.

a. Determine position, direction, and magnitude of the optimal (minimum) velocity variation that injects the spacecraft into a hyperbolic path, with hyperbolic excess velocity $v_\infty = 2 \text{ km/sec}$.

b. Calculate the limiting value of the true anomaly (at infinite distance).

8. A satellite is placed on a low Earth circular orbit, with radius of 7000 km and inclination of 20 deg. The final orbit is geosynchronous, with inclination of 5 deg. Determine

a. position, direction, and magnitude of the two (near-optimal) velocity changes needed to perform the orbit transfer;

b. the time of flight to complete the transfer at hand.

9. A spacecraft is placed in a polar Earth orbit, with apoapse and periapse altitudes of 2000 km and 400 km, respectively. The RAAN Ω equals 10 deg, and the apogee is located over the North pole.

a. Calculate the semimajor axis, eccentricity and argument of perigee

A first velocity change is applied, for the purpose of reducing the apogee altitude from 2000 km to 400 km.

b. Determine the optimal direction and magnitude of the velocity change, as well as the point where it must occur.

A second velocity variation is applied, in order to change the orbit inclination, from the initial value to the new value, equal to 100 deg. The remaining orbit elements are not altered. Determine

c. Magnitude, direction of this second velocity change, as well as the point where this must occur.

d. The maximal and minimum latitude flown by the spacecraft.

10. At the initial time t_0, set to 0, a satellites is placed on an Earth polar circular orbit, with the following orbit elements

$$R = 7000 \text{ km} \qquad \Omega = 10 \text{ deg} \qquad \theta_t(t_0) = 0 \text{ deg}$$

where R and Ω denote the orbit radius and RAAN, whereas θ_t is the argument of latitude. Determine

a. the time t_1 $(>t_0)$ at which the satellite flies over the North pole.

At t_1 a velocity change occurs, it is tangent to the trajectory and in the same direction of motion; the satellite flies again over the North pole 110 minutes after the velocity variation, at $t_2 = t_1 + 110 \text{ min}$. Determine

b. the magnitude of the velocity change;

c. the orbit eccentricity after the impulse.

11. At time t_0, a spacecraft is at the descending node, at geographical longitude of 110 deg, and travels a circular orbit, at altitude of 700 km, with inclination of 98.2 deg; the Greenwich sidereal time at t_0 equals 10 deg.

a. Calculate the right ascension of the ascending node (RAAN), Ω_0.

A single-impulse maneuver is applied, to change the RAAN to the (new) value Ω_f = 310 deg, without altering the inclination.

b. Identify the point where this velocity change is applied and calculate its magnitude.

c. Prove that this Δv only depends on $(\Omega_f - \Omega_0)$ (and not on Ω_f and Ω_0 separately).