# Spaceflight Mechanics

**Exercise set 4**

1. Calculate the synodic period of the inner planets (Venus and Mercury) relative to that of the Earth ( $\mu_{MER} = 22030 \text{ km}^3/\text{sec}^2$ , $\mu_{VEN} = 324900 \text{ km}^3/\text{sec}^2$ , $\mu_{SUN} = 132712 \cdot 10^6 \text{ km}^3/\text{sec}^2$ $r_{MER} = 57.91 \cdot 10^6 \text{ km}$ , $r_{VEN} = 108.2 \cdot 10^6 \text{ km}$ ). Then, calculate the spheres of influence of Venus and Mercury.

2. An Earth-to-Mars mission is planned, and employs a heliocentric Earth-to-Mars Hohmann transfer. The initial Earth orbit is circular, with radius 7000 km. The final Mars orbit is circular, with radius 5000 km. Calculate
   a. the magnitude of the velocity change along the Earth orbit;
   b. the magnitude of the single velocity change at Mars in order to inject the spacecraft into the desired orbit;
   c. the impact parameter (also termed aiming radius) at Mars;
   d. the Mars position relative to the Earth in order that Mars encounter can occur;
   e. the minimum wait time about Mars before the returning flight, which takes place again using a Hohmann heliocentric arc;
   f. the total mission time.

3. A spacecraft is launched on a mission to Mars, starting from a circular Earth orbit at altitude of 300 km. Periapse at Mars has radius of 6000 km. Calculate
   a. the magnitude of the velocity change along the initial Earth orbit.
   b. the two possible locations of the perigee (assuming that the Earth-centered hyperbola is coplanar with the Earth orbit around the Sun).
   c. the impact parameter (aiming radius) at Mars.

**4.** A spacecraft is in circular orbit about the Earth, with period of 90 minutes. Calculate

a. the magnitudes of the two velocity changes needed to inject the spacecraft into a circular orbit of radius equal to 384400 km;

b. the magnitude of the minimum velocity change needed to exit the Earth gravitational attraction;

c. the magnitude of the minimum velocity change needed to evade the solar system.

**5.** A space vehicle orbits the Sun, with a period of 300 sidereal days and eccentricity equal to 0.3. It is directed toward Venus.

a. Determine the hyperbolic excess velocity at arrival at Venus.

b. Under the assumption that the Venus-centered hyperbola has periapse radius of 10000 km, calculate position, direction, and magnitude of the single velocity change needed to inject the vehicle into a circular orbit of radius 10000 km.

**6.** A space vehicle orbits the Earth, with apoapse and periapse altitudes of 10000 km and 400 km. The Earth orbit is coplanar with the heliocentric arc, and has unspecified orientation.

a. Determine the optimal location, direction, and magnitude of the single velocity change needed to inject the vehicle toward Venus. In the heliocentric phase the the space trajectory is a Hohmann transfer (the Earth and Mars orbits are assumed coplanar and circular).

b. Instead of a single velocity change, consider the alternative three-impulse transfer, which includes two elliptic arcs with maximum apogee radius $r_A^{(max)}$. Determine the minimum value of $r_A^{(max)}$ in order that the three-impulse transfer be more convenient than the single-impulse transfer found at point a.

c. Calculate the magnitudes of the three velocity changes of the alternative three-impulse transfer, assuming $r_A^{(max)} = 300000$ km and $r_P^{(min)} = 6600$ km (minimum periapse radius of intermediate arcs)

**7.** A spacecraft departs Earth with a velocity perpendicular to the sun line on a flyby mission to Venus. Encounter occurs at a true anomaly in the approach trajectory of -30 deg. Periapse altitude over Venus is 300 km. Calculate

a. semimajor axis and eccentricity of the heliocentric ellipse;

b. the flight path angle along the heliocentric ellipse at Venus encounter;

c. the hyperbolic excess velocity at Venus, i.e. its magnitude and its components;

d. eccentricity, semimajor axis, and angular momentum of the planetocentric hyperbola

e. the deflection angle and the impact parameter (aiming radius) at Venus.

For a dark side approach, determine

f. the spacecraft velocity after flyby;

g. semimajor axis and eccentricity of the post-flyby heliocentric ellipse;

h. perihelion and aphelion radii of the post-flyby heliocentric ellipse;

i. the true anomaly along the post-flyby heliocentric ellipse.

For a sunlit approach, determine

l. the spacecraft velocity after flyby;

m. semimajor axis and eccentricity of the post-flyby heliocentric ellipse;

n. perihelion and aphelion radii of the post-flyby heliocentric ellipse;

o. the true anomaly along the post-flyby heliocentric ellipse.