1. A spacecraft travels an Earth elliptic orbit with periapse altitude equal to 622 km and focal distance 1500 km

(a) Obtain the semimajor axis, eccentricity, apoapse radius

(b) Obtain the (specific) angular momentum and energy

(c) Obtain the semiminor axis and semilatus rectum

2. A spacecraft travels a parabolic trajectory with semilatus rectum $p = 15000$ km, about Earth

(a) Obtain the (specific) angular momentum

(b) Calculate the periapse radius

(c) Depict the time histories of the radial and horizontal velocity components as functions of θ_* (true anomaly)

3. A spacecraft travels a hyperbolic trajectory about Earth. The two asymptotes of the hyperbola are orthogonal. The specific energy equals $\mathcal{E} = 5 \frac{\text{Km}^2}{\text{sec}^2}$

(a) Obtain the semimajor axis and eccentricity

(b) obtain the periapse radius and the angular momentum magnitude

(c) Depict the time histories of the radial and horizontal velocity components in terms of θ_*

(d) Evaluate the max value of the radial velocity

4. A spacecraft travels an orbit with instantaneous values of r, v_r, v_θ equal to

$$r = 8900 \text{ Km} \qquad v_r = -9.7 \frac{\text{Km}}{\text{sec}} \qquad v_\theta = 5.9 \frac{\text{Km}}{\text{sec}}$$

(a) Obtain the semimajor axis and eccentricity

(b) Evaluate the istantaneous true anomoly

(c) Portray the trajectory

5. A spacecraft has the following coordinates of position and velocity in a Cartesian frame (with center at Earth center)

$$X = 0 \text{ Km} \qquad Y = -10421 \text{ Km} \qquad V_x = 6.028 \frac{\text{km}}{\text{sec}} \qquad V_y = 0.550 \frac{\text{km}}{\text{sec}}$$

(a) Obtain the semimajor axis and eccentricity

(b) Evaluate the instantaneous true anomaly

(c) Portray the trajectory

Solutions

1. (a) $ea = 1500$ km

$a(1-e) = 7000$ km

(using $R_E = 6378$ km)

$\rightarrow$ $a = 8500$ km

$e = \frac{1500}{8500} = 0.176$

(b) $h = \sqrt{\mu a(1-e^2)} = 57293 \frac{km^2}{sec}$

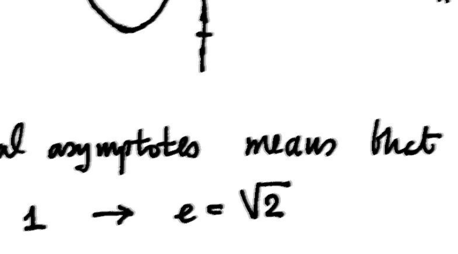
$\mathcal{E} = -\frac{\mu}{2a} = -23.45 \frac{km^2}{sec^2}$

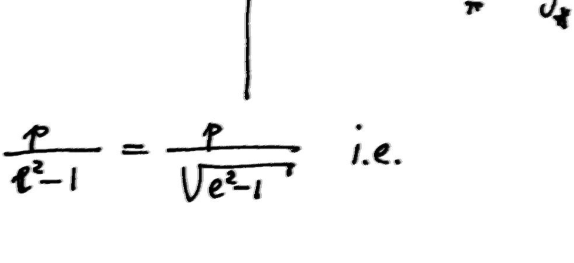
(c) $b = a\sqrt{1-e^2} = 8367$ km $\qquad p = a(1-e^2) = 8235$ km

2. (a) $\sqrt{p\mu} = h = 77324 \frac{km^2}{sec}$

(b) $r_p = \frac{p}{2} = 7500$ km

(c)





3. (a) Orthogonal asymptotes means that $\frac{p}{e^2-1} = \frac{p}{\sqrt{e^2-1}}$ i.e.

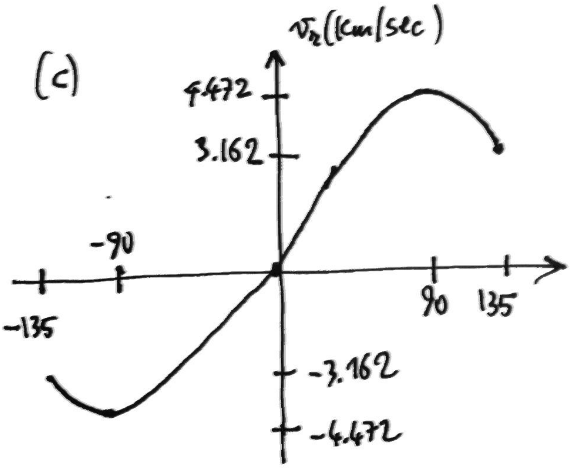
$\sqrt{e^2-1} = 1 \rightarrow e = \sqrt{2}$

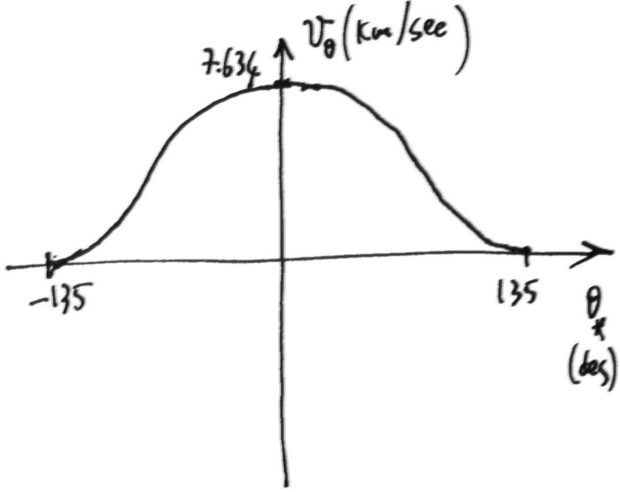
$\mathcal{E} = -\frac{\mu}{2a} \rightarrow a = -39860$ km

(b) $h = \sqrt{\mu a(1-e^2)} = 126049 \frac{km^2}{sec}$

$r_p = a(1-e) = 16511$ km

(c)





(d) $v_r^{(max)} = 4.472 \frac{km}{sec}$ at $\theta_* = \frac{\pi}{2}$

4. (a) $\mathcal{E} = -\frac{\mu}{r} + \frac{v_r^2 + v_\theta^2}{2} = 19.663 \frac{km^2}{sec^2} \rightarrow a = -\frac{\mu}{2\mathcal{E}} = -10136$ km (hyperbola)

$$h = \sqrt{\mu a(1-e^2)} = |\underline{r} \times \underline{v}| = r v_\theta$$

$$\rightarrow 1 - e^2 = \frac{(r v_\theta)^2}{\mu a} \rightarrow e = \sqrt{1 - \frac{(r v_\theta)^2}{\mu a}} = 1.297$$

(b) $r = \frac{p}{1 + e c_{\theta_*}} \rightarrow r e c_{\theta_*} = p - r \rightarrow c_{\theta_*} = \frac{p - r}{r e} =$

$$v_r = \sqrt{\frac{\mu}{p}}\, e s_{\theta_*} \rightarrow s_{\theta_*} = \frac{v_r}{e}\sqrt{\frac{p}{\mu}} =$$

$$\theta_* = 2\,\mathrm{atan}\frac{s_{\theta_*}}{1 + c_{\theta_*}} = -1.743 \simeq -99.9 \text{ deg}$$

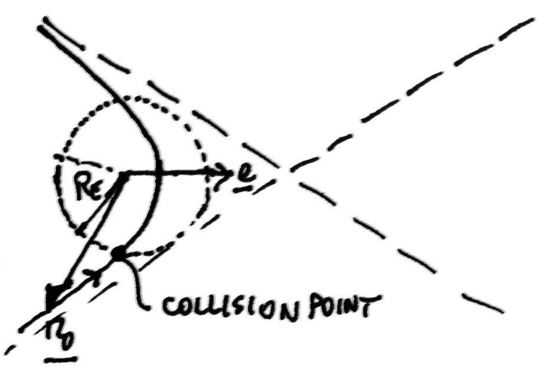
$r_p = 3011$ km

↓

periapse inside the Earth

↓

spacecraft on a collision course



(a) $\mathcal{E} = -\dfrac{\mu}{r} + \dfrac{v^2}{2} = -19.930 \dfrac{km^2}{sec^2} \rightarrow a = 10000$ km

$h = |\underline{r} \times \underline{v}| = 62817.8 \dfrac{km^2}{sec}$

$e = \sqrt{1 - \dfrac{h^2}{\mu a}} = 0.100$

(b) $v_r = \dfrac{X V_x + Y V_y}{r_0} = -0.550 \dfrac{km}{sec}$

$$\left.\begin{aligned} C_{\theta_*} &= \frac{p-r}{re} = -0.5 \\ S_{\theta_*} &= \frac{v_r}{e}\sqrt{\frac{P}{\mu}} = -\frac{\sqrt{3}}{2} \end{aligned}\right\} \rightarrow \theta_* = -120 \text{ deg}$$

(c)