Spaceflight Mechanics
Exercise Set M (Miscellaneous)

A.A. 2022-2023

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Data

Sidereal day	$\mathcal{T}_{sid} = 86\,164\,\text{s}$
Solar day	$\mathcal{T}_{sol} = 86\,400\,\text{s}$
Earth grav. param.	$\mu_{\oplus} = 398\,600.4\,\text{km}^3/\text{s}^2$

Exercise 1

A satellite is injected into a sub-orbital trajectory with perigee radius equal to 500 km and apogee radius 20 000 km. Assume no drag.

- Evaluate semi-major axis and eccentricity
- Evaluate velocity and flight path angle at departure and arrival point (on the ground)
- Evaluate the range and the travel time
- Evaluate the velocity increment (ΔV) required to put the satellite circularize the orbit at the apocenter altitude.

Results
$a = 10\,250\,\text{km}$, $e = 0.9512$, $V = 9.279\,\text{km}$, $\gamma = 70.53\,\text{deg}$,
$s = 6026\,\text{km}$, $\Delta t = 156.27\,\text{min}$, $\Delta V = 3.478\,\text{km/s}$

Exercise 2

A satellite is flying on an Molniya orbit, with Period $\mathcal{T} = 0.5\mathcal{T}_{sol}$, pericenter altitude $h_p = 1023\,\text{km}$, inclination $i = 63.4\,\text{deg}$, RAAN $\Omega = 60\,\text{deg}$, argument of pericenter $\omega = -90\,\text{deg}$.
Evaluate:

a) position and velocity in cartesian components at the true anomaly 18 deg.

b) absolute Longitude and latitude of the subsatellite point

c) flight path angle γ and east-heading angle ζ at that anomaly

Results
$\boldsymbol{r} = [3953.8,\ 413.2,\ -6425.2]^T\,\text{km}$, $\boldsymbol{v} = [3.999,\ 8.477,\ 1.547]^T\,\text{km/s}$
$\lambda_a = 5.97\,\text{deg}$, $\phi = -58.25\,\text{deg}$, $\gamma = 7.504\,\text{deg}$, $\zeta = 31.67\,\text{deg}$

Exercise 3

Solve the Hohmann transfer between two circular orbits of period 90 min and 600 min, respectively.
Results
$\Delta V_1 = 1.926\,\text{km/s}$, $\Delta V_2 = 1.383\,\text{km/s}$, $\Delta V_{tot} = 3.310\,\text{km/s}$

Exercise 4

A satellite is on circular orbit or radius 6600 km At time $t_0 = 0$, the satellite is on the vertical of Cape Canaveral (28.5 deg N, 80.5 deg W) and the velocity is directed toward East.

Part A: Design a mission to inject the satellite into a Geosyncronous (GEO) orbit, considering the following mission plans:

- one change of plane, separated maneuvers (S)
  LEO($i = i_{LEO}$) $\rightarrow$ GTO($i = i_{LEO}$) $\rightarrow$ GTO($i = 0$) $\rightarrow$ GEO($i = 0$)
- one change of plane, combined maneuvers (C)
  LEO($i = i_{LEO}$) $\rightarrow$ GTO($i = i_{LEO}$) $\rightarrow$ GEO($i = 0$)

Part B: Next, introducing a waiting orbit (WO) with apocenter on the GEO orbit, design a mission plan which allows to phase the satellite in GEO so that it belongs to the same meridian as Cape Canaveral. LEO($i = i_{LEO}$) $\rightarrow$ GTO($i = i_{GTO}$) $\rightarrow$ WO($i = i_{WO}$) $\rightarrow$ GEO($i = 0$).

Results
$r_{GEO} = 42\,164\,\text{km}$
Part A: $\Delta V_{tot}^S = 4.7107\,\text{km/s}$; $\Delta V_{tot}^C = 4.2833\,\text{km/s}$;

Exercise 5

Consider an Earth-Mars transfer. Assuming the heliocentric transfer is a Hohmann transfer, evaluate

- the transfer duration Δt and the phasing angle (γ_1) between Earth and Mars at departure
- the mission ΔV and the excess of hyperbolic velocity v_{∞_H}

departing from a circular orbit around the Earth for a one-way mission towards Mars. It has a structural coefficient $\epsilon = \frac{m_s + m_p}{m_s} = 7$) and specific impulse $I_{sp} = 450\,\text{s}$

Now, consider an increase of 1% of the excess of hyperbolic velocity v_∞ with respect to the Hohmann transfer, and evaluate

- the new excess of hyperbolic velocity v_∞
- the launch window duration Δt_{LW}

Results
$\Delta t = 256.62\,\text{days}$, $v_{\infty_h} = 2.946\,\text{km/s}$, $\gamma_{1_H} = 44.3\,\text{deg}$
$v_{\infty_1} = 2.975\,\text{km/s}$, $\gamma_{1'} = 45.16\,\text{deg}$, $\gamma_{1''} = 42.41\,\text{deg}$, $\Delta t_{LW} = 5.94\,\text{days}$

Exercise 6

Consider a spacecraft departing from the Earth for a one-way mission towards Mars. The heliocentric transfer is a Hohmann transfer. Evaluate the cost to insert the spacecraft in a capture orbit of period 1 Martian Day ($T_{\male} = 88\,775\,\mathrm{s}$) using

- a one-impulse strategy
- a two-impulse strategy
- a three-impulse strategy

Assume as minimum radius $r_{min} = 3597\,\mathrm{km}$ (= altitude 200 km) and maximum radius $r_{max} = 200\,000\,\mathrm{km}$.

Results
$\Delta V_{1-imp} = 1.9007\,\mathrm{km/s}$, $\Delta V_{2-imp} = 1.7085\,\mathrm{km/s}$, $\Delta V_{3-imp} = 1.3293\,\mathrm{km/s}$.

Exercise 7

Consider a spacecraft departing from the Earth for a one-way mission towards Mars. The heliocentric transfer is a Hohmann transfer. Evaluate the maximum theoretical inclination of the spacecraft orbit that can be achieved using a single flyby. Compare this results with the case of a flyby of minimum altitude $r_{min} = 3597\,\mathrm{km}$, that is, altitude 200 km.

Results
$\Delta i_{th} = 6.3032\,\mathrm{deg}$, $\Delta V_{r_{min}} = 6.2827\,\mathrm{deg}$.