Spaceflight Mechanics

Academic year 2023/2024

FINAL EXAM INFO

The final exam for the course Spaceflight Mechanics takes place in a single day and is composed of two parts:

1. **Theoretical questions** (T), with no book, note or electronic device allowed. This part includes 4 questions, to answer in 1.5 hours:
    a. 3 questions on orbital mechanics (chapters 2 through 8); these 3 questions include
        - 1 question on Chapter 5 (orbital motion in multibody environments),
        - 1 question on Chapter 8 (orbit perturbations), and
        - 1 question on a subject of the remaining chapters;
    b. 1 question on attitude dynamics (chapters 9,10,11)
2. **Numerical exercises** (N), with books and notes allowed. Electronic devices are allowed as well, provided that they are not capable of connecting to the web (no cell phone, laptop, or tablet is allowed). This part includes 3 exercises, to solve in 2 hours:
    a. 2 exercises on orbital mechanics, which include
        - 1 exercise on Chapter 4 (interplanetary trajectories), and
        - 1 exercise on a subject of Chapters 2,3,5,6,7,8
    b. 1 exercise on attitude dynamics (chapters 9 and 11)

For the final grade, the theoretical questions and the numerical exercises have the same weight (50% each). Moreover, the final grade is given by weighting the two main parts of the course:

(A) Orbital mechanics (chapters 2 through 8): 75%
(B) Attitude dynamics (chapters 9 through 11): 25%

Clarifications and explanations can be requested during appointments, to set by emailing the Course Instructors (mauro.pontani@uniroma1.it and alessandro.zavoli@uniroma1.it).

Booking the exam on Infostud by the due date is mandatory, with no exception. The exam is not allowed without booking. In case of any question or issue, please email the course Instructor at mauro.pontani@uniroma1.it before the end of the booking period.

COURSE PROGRAM

In the following, the program of the course is outlined. Most sections and subsections have letters T and/or N associated with them.

- T means that the content of the section (or subsection) may be requested in the theoretical questions;
- N means that the content of the section (or subsection) may be needed for the solution of the numerical exercises;
- T&N means that the content of the section (or subsection) may be requested in the theoretical questions and may be also useful for the solution of the numerical exercises.

Some sections (and subsections) are denoted neither with T nor with N. These sections include contents needed for a deep understanding of subsequent subjects.

Chapter 1 includes some math fundamentals, which are a prerequisite both for understanding all the course contents and for the numerical solution of exercises.

A few subsections are denoted with "(*outline*)", meaning that only a general overview was given in class.

- **Chapter 1. Math fundamentals**
  - Vector differentiation
  - Transport theorem
  - Elementary rotations
  - Composite rotations

- **Chapter 2. Keplerian trajectories**
  - Fundamental principles……..……..……..T
  - Two-body problem…….……T
    - Restricted two-body problem……..…..…….T
  - Gravitational potential…….……T
    - Spherical mass distribution……..……T
  - First integrals…….……..T
    - Angular momentum…….……T
    - Eccentricity vector…….…..T
  - Position and velocity……...N
    - Polar equation……...…….…….N
    - Cartesian equation……..….N

- Velocity……T&N
- Cosmic velocities……T&N

- Energy (per mass unit)……T&N
- Position in time……T
  - Eccentric anomaly (ellipses)……T&N
  - Kepler's equation (ellipses)……T&N
  - Solution for hyperbolas……N
- Special Keplerian trajectories……T
  - Circular orbits……T
  - Geosynchronous orbits……T
  - Ballistic trajectories……T
  - Rectilinear trajectories……T
- General classification of Keplerian orbits……T
  - Different Keplerian trajectories depending on position and velocity……T
- Representations in three dimensions……T
  - Earth-Centered-Inertial Frame (ECI)……T&N
  - Rotating frame (LVLH) ……T&N
  - Orbit elements……T&N
  - Cartesian coordinates……N
  - Spherical coordinates……N
  - Conversions among Cartesian, spherical coordinates, orbit elements……N
- Ground track……T&N
  - Latitude limits……T&N
  - Symmetry properties……T&N
  - Geosynchronous orbits……T&N
  - Geostationary orbits……T
  - Motion along the ground track……T
  - Examples of satellite ground tracks……T

❖ Chapter 3. Impulsive orbit transfers

- Impulsive thrust approximation……T
- Effect of velocity impulses……N
- Globally optimal transfers between coplanar circular orbits (*outline*)……T&N[1]
  - Hohmann transfer……T&N
  - Bielliptic transfer……T&N
  - Hohmann transfer vs. bielliptic transfer……T
- Three-dimensional orbit transfers……T
  - In-plane velocity changes……T
  - Out-of-plane velocity changes……T
  - Near-optimal LEO-GEO transfer……N
- Globally optimal transfers from elliptic orbit to hyperbolic path……T&N

[1] The complete proof on the global optimality of the Hohmann transfer is not being requested

[2] Only the overall effect of aerodynamic drag may be requested, not the analytical steps needed to derive the exact perturbing acceleration

[3] Only the definition and expression of the solar radiation pressure may be requested, not the analytical steps needed to derive the exact perturbing acceleration

[4] The derivation of the position vectors of the Moon and the Sun are not being requested

❖ **Chapter 9. Rigid Body Kinematics**

- Rigid-body-model of a spacecraft…….……..T
- Rotation matrix (direction cosine matrix)…….……..N
  - Kinematics equations……..N
  - Relative attitude…….T&N
- Sequences of angles……..……...……...…T&N
  - Singularities with three angles……...……...…..T[5]
  - Angles from rotation matrix……..…..N
  - Kinematics equations……..…N
- Principal axis and angle……..…….……T&N[6]
  - Proof 2 of Euler's theorem……..N
  - Principal axis and angle from rotation matrix…….N
  - Remarks…….T&N
- Euler parameters (quaternions)……..T&N
  - Basic properties…….T&N
  - Rotation matrix and Euler parameters……...…….N
  - Kinematics equations……..N
  - Relative quaternion using quaternion algebra……...T&N
- Comparison: Euler parameters vs. Sequences of angles.....T

❖ **Chapter 10. Complements of Newton mechanics**

- Systems of particles…….T
  - Linear momentum……...……T
  - Kinetic energy……...……..T
  - Angular momentum…….T
- Continuous systems…….T
  - Linear momentum……...……T
  - Angular momentum…….T

❖ **Chapter 11. Fundamentals of rigid body dynamics**

- Introduction to Rigid body dynamics……...……...T
  - Angular momentum……...…T&N
  - Inertia dyad……T&N
  - Properties of the inertia matrix…….……T&N[7]
  - Inertia matrix in different frames……..T&N
  - Principal axes of inertia……..N
  - Parallel axis theorem……..…T&N

[5] Explicit expressions of the rotation matrices are not being requested

[6] In the theoretical part, only the statement of the Euler's theorem may be requested, not the proof

[7] In the theoretical part, only the statements of the properties may be requested, not the related proofs

    - Inertia matrix of symmetric spacecraft…..……T&N
    - Euler equations of attitude dynamics..……..T&N
    - Kinetic energy……...T&N
- Torque-free motion
    - Axisymmetric body with no external torque……...……..T&N
    - General body with no external torque……T
    - Equilibrium solutions……...……T&N
    - Stability of pure spin……...…….....T&N
    - Energy and momentum integrals……T
    - Nutation of axisymmetric bodies.……..……...…..N
- Attitude maneuvers of spinning satellites……T
    - Spinup and spindown maneuvers……...…...T&N
    - Impulsive attitude maneuvers……..…T
    - Reorientation maneuvers……N

- **Exercise set 1 (Keplerian trajectories)**
- **Exercise set 2 (Keplerian trajectories)**
- **Exercise set 3 (Impulsive orbit transfers)**
- **Exercise set 4 (Interplanetary trajectories)**
- **Exercise set 5 (Relative orbit motion** and **Orbit Perturbations)**
- **Exercise set 6 (Rigid body kinematics** and **Eulerian mechanics)**
- **Exercise set 7 (Rigid body kinematics** and **Eulerian mechanics)**
- **Additional exercises – set 1**
- **Additional exercises – set 2**
- **Exam Test (part 2 – Numerical exercises)**

Further exercises can be found in the recommended textbooks, i.e.

(a) Prussing and Conway, *Orbital Mechanics* (chapters 1,2,3,7,9,10,11);

(b) Curtis, *Orbital Mechanics for Engineering Students* (chapters 2,3,4,6,7,8,9,10,12).