SOLUTIONS TO EXERCISE SETS

EXERCISE SET 2

1

(a) $\theta_* = 163.9$ deg

(b) $r = 38918$ Km $\quad v_r = 0.877 \frac{km}{sec} \quad v_\theta = 2.023$ km/sec

2

(a) $a = 16578$ km $\quad e = 0.591 \quad p = 10785$ km

$h = 65565.9 \frac{km^2}{sec} \quad \mathcal{E} = -12.022 \frac{km^2}{sec^2}$

(c) Velocity is max at periapse

Velocity is min at apoapse

At periapse and at apoapse velocity has the only horizontal component (i.e. $v_r = 0$)

(d) $\theta_* = acos(-e)$ where γ is max

$$\gamma_{max} = atan \frac{e}{\sqrt{1-e^2}} = 36.2 \text{ deg}$$

3

(a) 2 solutions exist for the possible eccentricity, i.e.

$e_1 = 0.450$

$e_2 = 0.379$

(b) $a_1 = 5449$ km ; $e_1 = 0.450$

$a_2 = 5449$ km ; $e_2 = 0.379$

(c) $h_{max,1} = a_1(1+e_1) - R_E = 1524$ km

$h_{max,2} = a_2(1+e_2) - R_E = 1135$ Km

(d) $\Delta t_1 = 33.95$ min

$\Delta t_2 = 30.63$ min

4 (a) $\Delta t = \sqrt{\frac{a^3}{\mu}}\left(\frac{\pi}{2} - e\right)$

(b) As $e \to 0$: $\Delta t = \frac{\pi}{2}\sqrt{\frac{a^3}{\mu}} = \frac{T}{4}$ $\quad T = 2\pi\sqrt{\frac{a^3}{\mu}}$

i.e. over a circular orbit the spacecraft travels from periapse to crossing the semilatus rectum in a quarter of the orbital period

5 (a) $a = 21872$ km $\quad e = 0.650 \quad i = 144.5$ deg

$\Omega = 219.5$ deg $\quad \omega = 243.7$ deg

(b) $E = 42.4$ deg $\quad M = 17.3$ deg $\quad \theta_* = 80.3$ deg

(c) $X = -3657$ km $\quad Y = -10047$ Km $\quad Z = -3892$ Km

$V_x = -6.596\ \frac{Km}{sec} \quad V_y = -2.334\ \frac{Km}{sec} \quad V_z = 1.698\ \frac{Km}{sec}$

6 (a) $a = 9505$ km $\quad e = 0.079 \quad i = 7.0$ deg

$\Omega = 4.0$ deg $\quad \omega = 272.8$ deg

(b) $\theta_* = 128.4$ deg $\quad E = 124.8$ deg $\quad M = 121.1$ deg

(c) $r = 9932$ Km $\quad \lambda_a = 45.0$ deg $\quad \phi = 4.6$ deg

$\gamma = 3.7$ deg $\quad v = 6.191\ \frac{km}{sec} \quad \zeta = 5.3$ deg

EXERCISE SET 3

$\boxed{1}$ (a) $\theta_* = 0$ (b) $e = 0.44$

$\boxed{2}$ (a) $\theta_* = \frac{\pi}{2}$ (b) $e = 0.2$

$\boxed{3}$ $b = \pm\sqrt{2}$

$\boxed{4}$ $b = \pm 1$

$\boxed{5}$ $\Delta v_1 = 1.167 \frac{km}{sec}$ $\Delta v_2 = 0.979 \frac{km}{sec}$

$\boxed{6}$ (a) $\Delta v_{tot} = 4.015 \frac{km}{sec}$ (b) $\Delta v_{tot} = 3.878 \frac{km}{sec}$ (c) $\Delta v_{tot} = 3.729 \frac{km}{sec}$

(d) $\Delta t^{(H)} = 48.13$ hrs, $\Delta t^{(BE)} = 473.8$ hrs, $\Delta t^{(BP)} \to \infty$

$\boxed{7}$ (a) Impulse at periapse, same direction of the motion, (horizontal direction), magnitude $\Delta v = 2.243 \frac{km}{sec}$

(b) $\theta_*^M = 159.0$ deg

$\boxed{8}$ (a) $\Delta v_1 = 2.337 \frac{km}{sec}$, $\Delta v_2 = 1.549 \frac{km}{sec}$

(b) $\Delta t = 319.6$ min

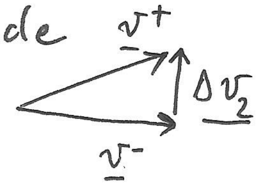
First impulse at node
Second impulse at opposite node
(1^{st} impulse in-plane, 2^{nd} combines in-plane and out-of-plane).

$\boxed{9}$ (a) $a = 7578$ km $\quad e = 0.106 \quad \omega = -\frac{\pi}{2}$

(b) Impulse at periapse, direction is tangential and opposite to motion, $\Delta v = 0.395$ km/sec

(c) Impulse at ascending or descending node

$\Delta v = 1.337 \frac{km}{sec}$



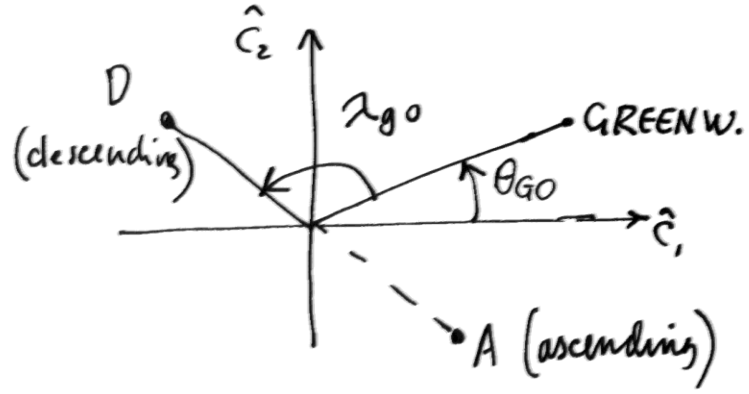
(d) $\phi_{max} = 80$ deg $= -\phi_{min}$

$\boxed{10}$ (a) $\Delta t = 24.3$ min

(b) $\Delta v = 0.294 \frac{km}{sec}$

(c) $e = 0.080$

$\boxed{11}$



(a) $\Omega_0 = \lambda_{g0} + \theta_{G0} - \pi = 300\ \text{deg}$

(b) Point at which impulse is applied satisfies

$$\begin{cases} X = R\left(C_{\theta_{t1}} C_{\Omega_1} - C_i \cdot S_{\theta_{t1}} S_{\Omega_1}\right) = R\left(C_{\theta_{t2}} C_{\Omega_2} - C_i \cdot S_{\theta_{t2}} S_{\Omega_2}\right) \\ Y = R\left(C_{\theta_{t1}} S_{\Omega_1} + C_i \cdot S_{\theta_{t1}} C_{\Omega_1}\right) = R\left(C_{\theta_{t2}} S_{\Omega_2} + C_i \cdot S_{\theta_{t2}} C_{\Omega_2}\right) \\ Z = R\, S_{\theta_{t1}} S_i = R\, S_{\theta_{t2}} S_i \end{cases}$$

From the 3^{rd}, $\theta_{t2} = \pi - \theta_{t1}$, this is inserted in the eqs for X and Y. These 2 equations are equivalent, thus only 1 is necessary, i.e.

$$C_{\theta_t}\left(C_{\Omega_1} + C_{\Omega_2}\right) + S_{\theta_t}\left(S_{\Omega_2} - S_{\Omega_1}\right) C_i = 0 \rightarrow \tan\theta_t = -\frac{C_{\Omega_1} + C_{\Omega_2}}{C_i\left(S_{\Omega_2} - S_{\Omega_1}\right)}$$

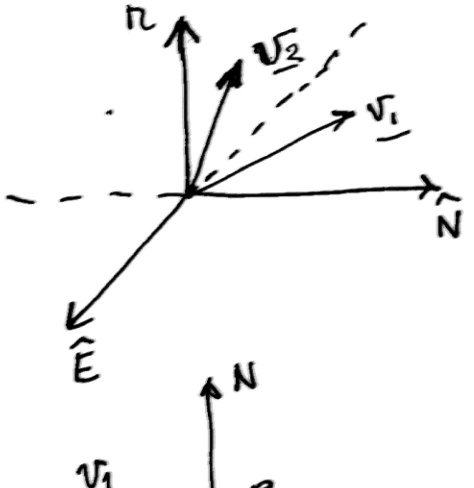
$$\rightarrow \theta_t^{(1)} = \operatorname{atan}\left(-\frac{C_{\Omega_1} + C_{\Omega_2}}{C_i\left(S_{\Omega_2} - S_{\Omega_1}\right)}\right) \ ; \ \theta_t^{(2)} = \theta_t^{(1)} + \pi$$

Once θ_t is found, one obtains

$\varsigma_1 = \operatorname{atan2}\left(S_{\theta_{t1}} S_i\ ,\ C_i\right)$ and $\varsigma_2 = \operatorname{atan2}\left(S_{\theta_{t2}} S_i\ ,\ C_i\right)$

$$\begin{cases} \varsigma_1 = 3.0552 = 175\ \text{deg} \\ \varsigma_2 = -3.0552 = -175\ \text{deg} \end{cases}$$

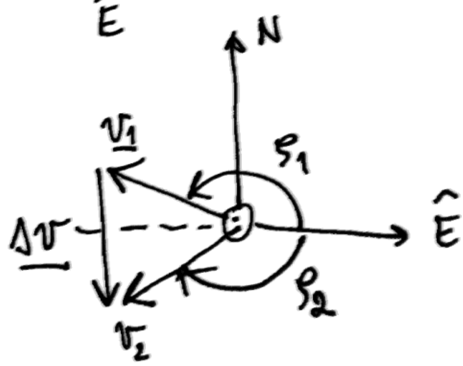
$$\Delta v = 2 v_1 \sin\left(\pi - \varsigma_1\right) = 1.295\ \frac{km}{s}$$



(c) Dependancy on (Ω_1, Ω_2) is in θ_t thus one must prove that θ_t depends on $(\Omega_2 - \Omega_1)$. Using

$$\begin{cases} \cos x + \cos y = 2\cos\frac{x+y}{2}\cos\frac{x-y}{2} \\ \sin x - \sin y = 2\sin\frac{x-y}{2}\cos\frac{x+y}{2} \end{cases}$$

$$\tan\theta_t = \frac{1}{C_i \tan\left(\frac{\Omega_2 - \Omega_1}{2}\right)}$$



Alternative proof is geometric, i.e. prove that geom of intersection depends on $(\Omega_2 - \Omega_1)$

EXERCISE SET 4

Exercise 1

$$T_{syn} = \frac{2\pi}{|n_1 - n_2|} = \begin{matrix} 115.9 \text{ days} & \text{for Earth and Mercury} \\ 582.6 \text{ days} & \text{for Earth and Venus} \end{matrix}$$

$$r_{SOI} = r_p \left(\frac{m_p}{m_s}\right)^{\frac{2}{5}} = \begin{matrix} 112412 \text{ km} & \text{for Mercury} \\ 616317 \text{ km} & \text{for Venus} \end{matrix}$$

Exercise 2

Earth - Mars transfer

$r_E = 149.6 \cdot 10^6$ km

$r_M = 227.9 \cdot 10^6$ km

$\mu_S = 132.7 \cdot 10^9 \frac{km^3}{sec^2}$ $\quad \mu_E = 398600 \frac{km^3}{sec^2}$ $\quad \mu_M = 42828 \frac{km^3}{sec^2}$

(a) $$a_T = \frac{r_E + r_M}{2} \qquad e_T = \frac{r_M - r_E}{r_M + r_E}$$

$$\Delta t_H = \pi \sqrt{\frac{a_T^3}{\mu_S}} = 258.82 \text{ days}$$

$$\Delta v_1 = \sqrt{\frac{\mu_S}{a_T}\frac{1+e_T}{1-e_T}} - \sqrt{\frac{\mu_S}{r_E}} = 2.943 \frac{km}{sec} = v_\infty^{(P1)}$$

$$\Delta v_2 = \sqrt{\frac{\mu_S}{r_M}} - \sqrt{\frac{\mu_S}{a_T}\frac{1-e_T}{1+e_T}} = 2.648 \frac{km}{sec} = v_\infty^{(P2)}$$

$$v_p^{(P1)} = \sqrt{\frac{2\mu_E}{r_p^{(P1)}} + \left[v_\infty^{(P1)}\right]^2} = 11.070 \frac{km}{sec}$$ velocity at periapse of outgoing hyperbola

$$\Delta v_1^{(P1)} = v_p^{(P1)} - \sqrt{\frac{\mu_E}{r_{0,E}}} = 3.524 \frac{km}{sec}$$ at injection from Earth.

(b) $$v_p^{(P2)} = \sqrt{\frac{2\mu_M}{r_p^{(P2)}} + \left[v_\infty^{(P2)}\right]^2} = 4.913 \frac{km}{sec}$$ velocity at periapse of Mars hyperbola

$$\Delta v_2^{(P2)} = v_p^{(P2)} - \sqrt{\frac{\mu_M}{r_{f,M}}} = 1.987 \frac{km}{sec}$$ at Mars injection

(c) $a^{(P2)} = -\dfrac{\mu_M}{\left[v_\infty^{(P2)}\right]^2} = -6108.4\ \text{km}$

$r^{(P2)} = a^{(P2)}\left[1 - e_{HYP}^{(P2)}\right] \rightarrow e_{HYP}^{(P2)} = 1.818$

$\chi^{(P2)} = -a^{(P2)}\sqrt{e_{HYP}^2 - 1} = 9278.1\ \text{km}$

(d) For Hohmann transfer

$\widetilde{\Delta\phi}_1 = \pi \qquad \Delta t_1 = 258.82\ \text{days}$

$\phi_{M,0} = \widetilde{\Delta\phi}_1 - n_M \Delta t_1 = 44.6\ \text{deg}$

initial angle of Mars relative to the Earth

(e) For a returning Hohmann transfer

$\widetilde{\Delta\phi}_2 = \pi \qquad \Delta t_2 = \Delta t_1$

$$t_W = \frac{\widetilde{\Delta\phi}_2 + \phi_{M,1} - \phi_{E,1} - n_E \Delta t_2}{n_E - n_M} + \Delta t_{SYN}$$

where $\Delta t_{SYN} = 780.2$ days (synodic period)

$t_W = 454.7$ days (waiting time)

(f) $\Delta t_{TOT} = \Delta t_1 + t_W + \Delta t_2 = 972.37$ days.

Exercise 3

$$\Delta v_1 = 2.943 \frac{km}{sec} = v_\infty^{(P1)}$$

(see previous exercise)

$$\Delta v_2 = 2.648 \frac{km}{sec} = v_\infty^{(P2)}$$

(a) $$v_p^{(P1)} = \sqrt{\frac{2\mu_E}{r_p^{(P1)}} - \left[v_\infty^{(P1)}\right]^2} = 11.316 \frac{km}{sec}$$

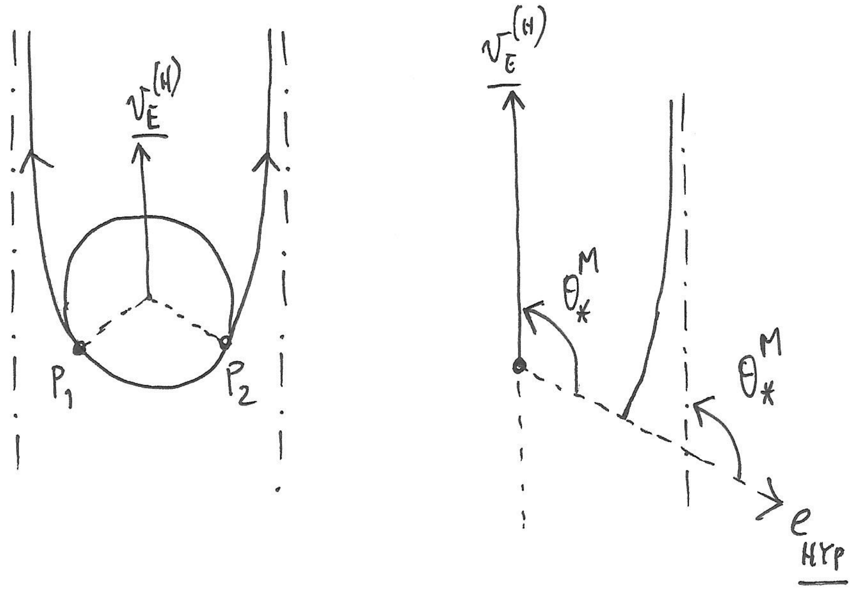
$$\Delta v_1^{(P1)} = v_p^{(P1)} - \sqrt{\frac{\mu_E}{r_0}} = 3.590 \frac{km}{sec}$$

(b) $$a^{(P1)} = -\frac{\mu_E}{\left[v_\infty^{(P1)}\right]^2} = -46008 \text{ km}$$

$$e_{HYP}^{(P1)} = 1 - \frac{r_p^{(P1)}}{a^{(P1)}} = 1.145$$ eccentricity of Earth hyperbola

$$\theta_*^M = \text{acos}\left(-\frac{1}{e_{HYP}^{(P1)}}\right) = $$

$$= 151.8 \text{ deg}$$



(c) $$v_\infty^{(P2)} = 2.648 \frac{km}{sec}$$

$$a^{(P2)} = -6108 \text{ km}$$

$$e_{HYP}^{(P2)} = 1.982$$

$$\chi = 10455 \text{ km}$$ (impact parameter or aiming radius)

Exercise 4

$$T = 2\pi\sqrt{\frac{a^3}{\mu_E}} \rightarrow a = \left[\left(\frac{T}{2\pi}\right)^2 \mu_E\right]^{1/3} = 6652.6 \text{ km} \rightarrow R_i = a$$

(a) Hohmann

$$\Delta v_1 = 3.113 \frac{km}{sec}$$

$$\Delta v_2 = 0.830 \frac{km}{sec}$$

$$\Delta v_{tot} = 3.943 \frac{km}{sec}$$

(b) $$\Delta v_{ESC} = (\sqrt{2}-1)\sqrt{\frac{\mu_E}{R_i}} = 3.206 \frac{km}{sec.}$$

(c) First, while exiting the Earth sphere of influence

$$\Delta v_1^{(H)} = (\sqrt{2}-1)\sqrt{\frac{\mu_S}{r_E}} = 12.337 \frac{km}{sec} \equiv v_\infty^{(E)}$$

Then

$$v_p^{(E)} = \sqrt{\frac{2\mu_E}{r_p^{(E)}} + \left[v_\infty^{(E)}\right]^2} = 16.494 \frac{km}{sec}$$

$$\Delta v_p^{(E)} = v_p^{(E)} - \sqrt{\frac{\mu_E}{R_i}} = 8.753 \frac{km}{sec.}$$

Exercise 5

(a) $v_\infty = 9.571 \; \frac{km}{sec}$

(b) $\Delta v = 6.812 \; \frac{km}{sec}$

Exercise 6

(a) $\Delta v = 2.008 \; km/sec$

(b) $r_{A,lim}^{(max)} = 127670 \; km$

$r_A^{(max)} > r_{A,lim}^{(max)} \Rightarrow$ 3-impulse transfer is more convenient

(c) $\Delta v_1 = 1.604 \; km/sec$ $\Delta v_3 = 0.399 \; km/sec$

$\Delta v_2 = 0.003 \; km/sec$

Exercise 7

(a) $a^{(H)} = 127.74 \cdot 10^6 \; km$ $e^{(H)} = 0.170$

(b) $\gamma = -4.2 \; deg$

(c) $v_\infty^{(PZ)} = 3.740 \; \frac{km}{sec}$

(d) $e_{HYP}^{(PZ)} = 1.273$ $a_{HYP}^{(PZ)} = -23283 \; km$ $h_{HYP}^{(PZ)} = 6.857 \cdot 10^4 \; \frac{km^2}{sec}$

(e) $\xi = 103.6 \; deg$

(f) $v_{r,+}^{(H)} = -1.773 \; \frac{km}{sec}$ $v_{\theta,+}^{(H)} = 31.740 \; \frac{km}{sec}$

(g) $a_+^{(H)} = 91.86 \cdot 10^6 \; km$ $e_+^{(H)} = 0.185$

(h) $r_{A,+}^{(H)} = 108.85 \cdot 10^6 \; km$ $r_{P,+}^{(H)} = 78.88 \cdot 10^6 \; km$

(i) $\theta_{*,+} = -165.6 \; deg$

(l) $v_{r,t}^{(H)} = 3.079 \; \frac{km}{sec}$ $\quad$ $v_{\theta,t}^{(H)} = 37.161 \; \frac{km}{sec}$

(m) $a_{t}^{(H)} = 124.6 \cdot 10^{6} \; km$ $\quad$ $e_{t}^{(H)} = 0.156$

(n) $r_{A,t}^{(H)} = 144 \cdot 10^{6} \; km$ $\quad$ $r_{P,t} = 105.19 \cdot 10^{6} \; km$

(o) $\theta_{*,t} = 36.7 \; deg$

- EXERCISE SET 5

[1] $\begin{cases} x_0 = \delta r_0 = 50 \text{ km} & R = R_E + 450 \text{ km} \quad \omega_R = \sqrt{\dfrac{\mu_E}{R^3}} \\ y_0 = R\,\delta\xi_0 \quad \text{where } \delta\xi_0 = 30 \text{ deg} \; ; & c = 3 \dfrac{\text{km}}{\text{sec}} \end{cases}$

(a) Chaser in circular orbit $\Rightarrow \begin{cases} \dot{y}_0^- = -\dfrac{3}{2}\omega_R x_0 \\ \dot{x}_0^- = 0 \end{cases}$

Interception occurs after $t_f = T_f = 60$ minutes $= 3600$ sec

First of all, $c_f = \cos(\omega_R t_f)$ and $s_f = \sin(\omega_R t_f)$ are evaluated

Then, the 2 conditions

$\begin{cases} x_f = 0 \\ y_f = 0 \end{cases}$ are enforced, using

$$\begin{cases} x_f = (4 - 3c_f)x_0 + \dfrac{s_f}{\omega_R}\dot{x}_0^+ + \dfrac{2}{\omega_R}(1 - c_f)\dot{y}_0^+ = 0 \\ y_f = 6(s_f - \omega_R t_f)x_0 + y_0 + \dfrac{2}{\omega_R}(c_f - 1)\dot{x}_0^+ + \left(\dfrac{4}{\omega_R}s_f - 3t_f\right)\dot{y}_0^+ = 0 \end{cases}$$

leading to

$$\begin{bmatrix} \dot{x}_0^+ \\ \dot{y}_0^+ \end{bmatrix} = \begin{bmatrix} \dfrac{s_f}{\omega_R} & \dfrac{2}{\omega_R}(1 - c_f) \\ \dfrac{2}{\omega_R}(c_f - 1) & \dfrac{4}{\omega_R}s_f - 3t_f \end{bmatrix}^{-1} \begin{bmatrix} -(4 - 3c_f)x_0 \\ -6(s_f - \omega_R t_f)x_0 - y_0 \end{bmatrix}$$

$\dot{x}_0^+ = 0.571 \dfrac{\text{km}}{\text{sec}} = \dot{x}_0^- + \Delta v_{x,1} \longrightarrow \Delta v_{x,1} = 0.571 \dfrac{\text{km}}{\text{sec}}$

$\dot{y}_0^+ = 0.034 \dfrac{\text{km}}{\text{sec}} = \dot{y}_0^- + \Delta v_{y,1} \longrightarrow \Delta v_{y,1} = 0.118 \dfrac{\text{km}}{\text{sec}}$

(b) For rendezvous, a second velocity change is needed at the final point, when C reaches T. At the end $\dot{x}_f^+ = 0$ and $\dot{y}_f^+ = 0$, whereas

$$\begin{cases} \dot{x}_f^- = 3\omega_R x_0 s_f + c_f \dot{x}_0^+ + 2 s_f \dot{y}_0^+ = -0.544 \dfrac{km}{sec} \\ \dot{y}_f^- = 6\omega_R [c_f - 1] x_0 - 2 s_f \dot{x}_0^+ + (4c_f - 3)\dot{y}_0^+ = 0.146 \dfrac{km}{sec} \end{cases}$$

$$\Delta v_{x,2} = \dot{x}_f^+ - \dot{x}_f^- = 0.544 \frac{km}{sec}$$

$$\Delta v_{y,2} = \dot{y}_f^+ - \dot{y}_f^- = -0.146 \frac{km}{sec}$$

(c) To find propellant consumption, Tsiolkovsky's law is used

$$\frac{m_f}{m_0} = \exp\left[-\frac{\Delta v_1 + \Delta v_2}{c}\right], \text{ where } \begin{cases} \Delta v_1 = \sqrt{\Delta v_{x,1}^2 + \Delta v_{y,1}^2} = 0.583 \dfrac{km}{sec} \\ \Delta v_2 = \sqrt{\Delta v_{x,2}^2 + \Delta v_{y,2}^2} = 0.563 \dfrac{km}{sec} \end{cases}$$

$$\rightarrow \quad m_{prop} = m_0 - m_f = 1589 \text{ kg}$$

(d) Hohmann transfer corresponds to 2 horizontal velocity changes and takes $\Delta t = \frac{\pi}{\omega_R}$, under the assumption of approximating the transfer time (along the Hohmann ellipse) as half of the orbital period of the reference orbit. In this case $T_f = \frac{\pi}{\omega_R}$

$$\begin{cases} c_f = \cos(\omega_R T_f) = -1 \\ s_f = \sin(\omega_R T_f) = 0 \end{cases} \quad \text{leading to}$$

$$x_f = (4+3)x_0 + \frac{2}{\omega_R} 2 \dot{y}_0^+ = 0 \quad \rightarrow \quad \dot{y}_0^+ = -\frac{7}{4}\omega_R x_0$$

$$\rightarrow \quad \Delta v_y^{(1)} = \dot{y}_0^+ - \dot{y}_0^- = \omega_R x_0 \left[-\frac{7}{4} + \frac{3}{2}\right] = -\frac{\omega_R x_0}{4} = -14 \frac{m}{sec}$$

This value of $\dot{y}_0^+$ is inserted into the equation for y_f:

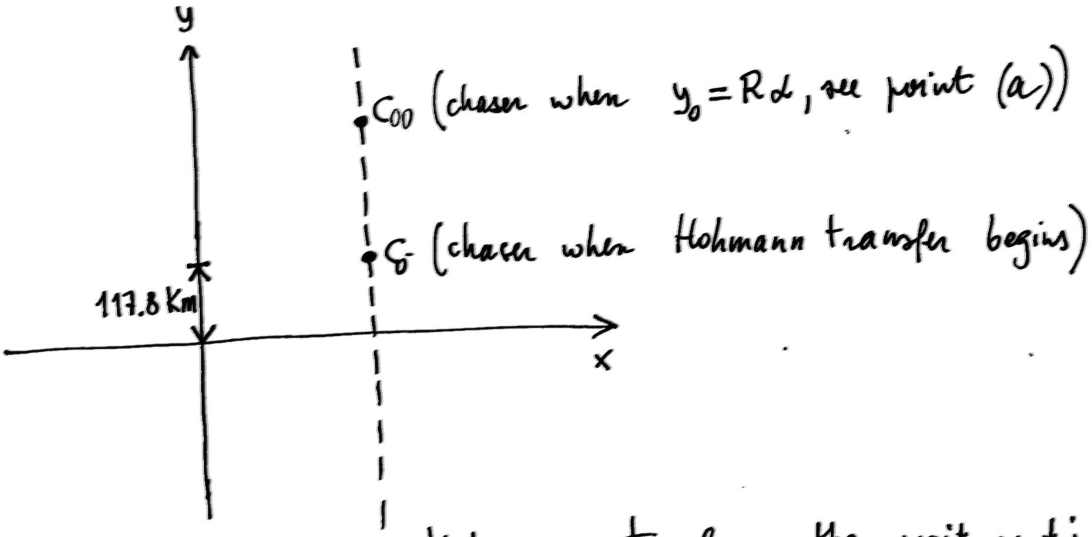
$$y_f = -6\pi x_0 + y_0 - 3\frac{\pi}{\omega_R}\dot{y}_0^+ =$$

$$= -6\pi x_0 + y_0 - \frac{3\pi}{\omega_R}\left(-\frac{7}{4}\omega_R x_0\right) =$$

$$= y_0 - \frac{3\pi}{4}x_0$$

In order that $y_f = 0$ at t_f y_0 must be selected such as $y_0 - \frac{3\pi}{4}x_0 = 0$, i.e.

$$y_0 = \frac{3\pi}{4}x_0 = 117.8 \text{ km}$$



Before starting the Hohmann transfer, the waiting time is

$$t_{wait} = \frac{y_0 - R\alpha}{-\frac{3}{2}\omega_R x_0} = 41198 \text{ sec}$$

velocity with which the line ⋮ is traveled

Therefore the Hohmann transfer starts when $y_0 = 117.8$ km corresponding to $\quad \delta\xi_0 = \dfrac{y_0}{R_{ref}} = 1.0 \text{ deg}$

(e) The first velocity change was already found,

$$\Delta v_y^{(1)} = -\frac{\omega_R}{4} x_0 = -0.014 \frac{\text{km}}{\text{sec}}$$

The second velocity change (for rendezvous) is such that

$$\begin{cases} \dot{y}_f^+ = 0 \\ \dot{x}_f^+ = 0 \end{cases} \quad \text{at} \quad \tau = \Delta t$$

The general expression for $\dot{y}$ is

$$\dot{y} = 6\omega_R (c-1) x_0 - 2s\dot{x}_0 + (4c-3)\dot{y}_0$$

evaluated at $\Delta t = \frac{\pi}{\omega_R}$ yields

$$\dot{y}_f^- = -12\omega_R x_0 - 7\dot{y}_0^+ = -12\omega_R x_0 - 7\left(-\frac{3}{2}\omega_R x_0 - \frac{1}{4}\omega_R x_0\right)$$

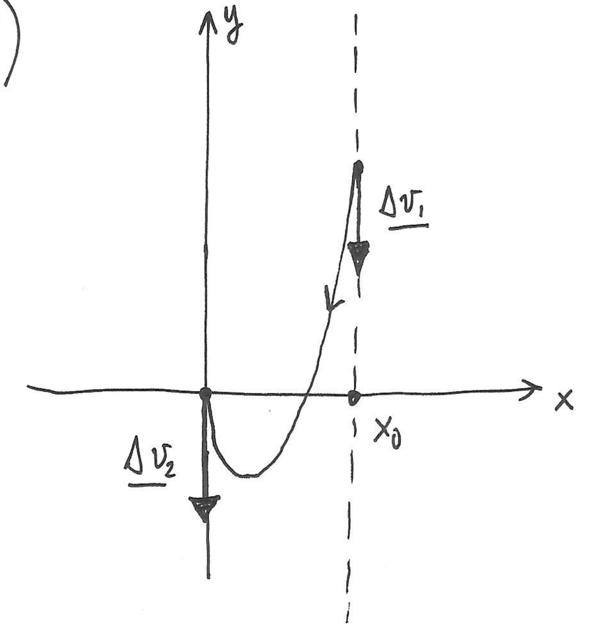
$$= -12\omega_R x_0 + \frac{49}{4}\omega_R x_0 = \frac{\omega_R x_0}{4}$$

Because $\dot{y}_f^+ = 0$ one obtains

$$\Delta v_y^{(2)} = \dot{y}_f^+ - \dot{y}_f^- = -\frac{\omega_R x_0}{4} = -0.014 \frac{\text{km}}{\text{sec}}$$

The second impulse is identical to the first

(f)



(g) Using the previously found relation

$$m_p = 46 \text{ kg}$$

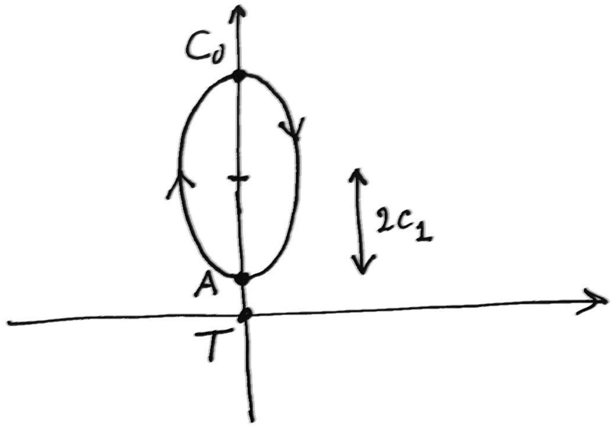
$\boxed{2}$

$$\begin{cases} x_0 = 0 \\ y_0 = 10 \text{ km } (= d) \end{cases} \qquad \begin{cases} \dot{x}_0^- = 0 \\ \dot{y}_0^- = 0 \end{cases} \qquad \begin{matrix} R = R_E + 700 \text{ km} \\ d_{min} = 30 \text{ m} \end{matrix}$$

Fundamental condition for a relative elliptic orbit is

$$\dot{y}_0^+ + 2\omega_R x_0 = 0 \quad \text{leading to} \quad \dot{y}_0^+ = 0 \quad \text{because } x_0 = 0$$

The semimajor axis of the relative ellipse has length $2c_1$ and point A (closest approach to T) is 30 m far from T



$$\rightarrow 2c_1 = \frac{d - d_{min}}{2} \rightarrow c_1$$

T = space debris
C_0 = spacecraft at $t_0 = 0$

However

$$c_1 = \sqrt{\left(3x_0 + \frac{2\dot{y}_0^+}{\omega_R}\right)^2 + \left(\frac{\dot{x}_0^+}{\omega_R}\right)^2} \quad , \text{in general}$$

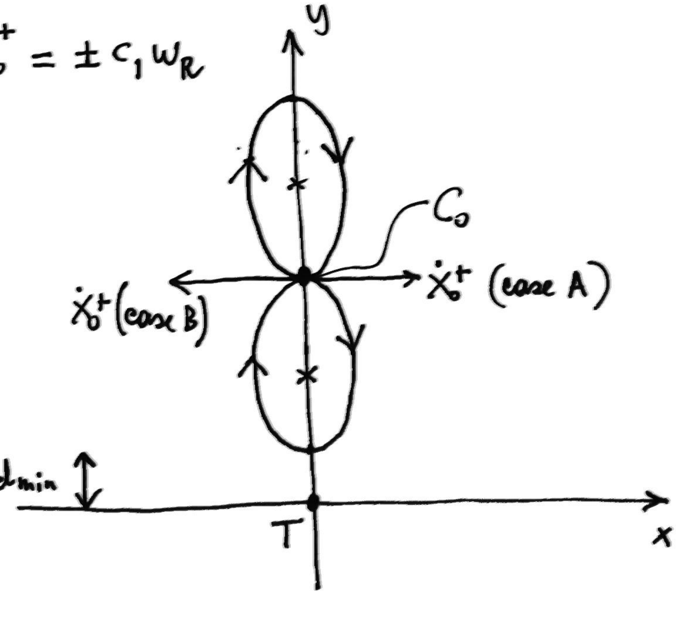
leading to

$$\left|\frac{\dot{x}_0^+}{\omega_R}\right| = c_1 \quad \text{i.e.} \quad \dot{x}_0^+ = \pm c_1 \omega_R$$

case A: lower ellipse is traveled

case B: higher ellipse is traveled

→ correct solution is A



thus $\dot{x}_0^+ = \Delta v_x = c_1 \omega_R = 2.65 \frac{m}{sec}$

$\boxed{3}$ After the velocity change

(a) $$\begin{cases} x(\tau) = \frac{2}{\omega_R}(1-c)\,\dot{y}_0^+ \\ y(\tau) = y_0 + \left(\frac{4S}{\omega_R} - 3\tau\right)\dot{y}_0^+ \end{cases}$$ because $x_0 = 0$ and $\dot{x}_0^+ = 0$

Moreover $\omega_R \tau_f = 2\pi$, therefore

$$x(\tau_f) = 0$$

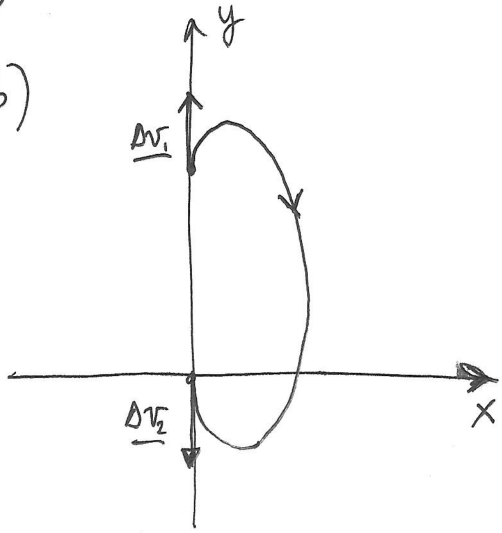
$$y(\tau_f) = y_0 - 3 \cdot 2\pi \frac{\dot{y}_0^+}{\omega}$$

Because $y(\tau_f)$ must equal 0, one gets $\dot{y}_0^+ = \frac{\omega_R}{6\pi} y_0 = 0.56 \frac{m}{sec}$

The second velocity change leads to obtaining zero final relative velocity

$$\dot{y}_f^- = \left(4\cos(\omega_R \tau) - 3\right)\dot{y}_0^+ \underset{\substack{\uparrow \\ \omega_R \tau = 2\pi}}{=} \dot{y}_0^+$$

(b)



Therefore $\Delta v_{1,y} = \dot{y}_0^+ = 0.56 \frac{m}{sec}$

$\Delta v_{2,y} = -\dot{y}_0^+ = -0.56 \frac{m}{sec}$

$\boxed{4}$ (a) From the equation of i of sunsynchronous orbits

$i = 98.0$ deg

(b) As a first step ρ (atmospheric density) is evaluated at 650 km of altitude

$$\rho = \rho_0 \, e^{-\frac{h-h_0}{H}} \qquad \rho_1 = \rho_0 e^{-\frac{h_1 - h_0}{H}}$$

$h_1 = R_E + 700$ km

$h_0 = R_E + 600$ km

$$\ln \frac{\rho_1}{\rho_0} = \frac{h_0 - h_1}{H} \rightarrow H = \frac{h_0 - h_1}{\ln \frac{\rho_1}{\rho_0}} \Rightarrow \rho(650\,km + R_E) = 7.249 \cdot 10^{-14} \frac{kg}{m^3}$$

Using the simplified formula for a_{fin},

$$a_{fin} = \left\{ a_{ini}^{1/2} - \frac{C_D S}{2m} \rho \sqrt{\mu} \left(t_{fin} - t_{ini} \right) \right\}^2 = 7027.9 \text{ km}$$

where $t_{fin} - t_{ini} = 30$ days

The semimajor axis has decreased by 198 m

$\boxed{5}$ Using the double-average equation for the precession effect due to the 3rd body, one obtains

$$T_{prec} = \frac{8\pi}{3} \frac{h r_{12}^3}{\mu_2 R^2} \left| \left(\hat{h}^T \hat{h}_{3B} \right) \right|^{-1}$$

where $R = 42164$ km for a geostationary orbit.

Moreover $\hat{h}^T \hat{h}_{3B} = \cos \epsilon$ (ϵ = ecliptic obliquity)

(a) $T_{prec,\odot} = 534.05$ years

(b) $T_{prec,☾} = 245.22$ years